Correlations for calculating the transport and thermodynamic properties of lead-bismuth eutectic

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Abstract

The paper presents recommended correlations for calculating the thermodynamic and transport properties of Pb-Bi eutectic (44.5% Pb + 55.5% Bi), namely: density, dynamic viscosity, specific heat, thermal conductivity, surface tension, specific electrical resistance, and local speed of sound as a function of temperature. These correlations are based on calculated data presented in 39 experimental studies performed in our country and abroad and published during the period from 1923 to 2015. The authors had information on 1103 experimental points; however, a direct assessment was performed on 1076 points. The main difficulty in processing the data was that the experiments considered in the work were performed at different times using a variety of measurement methods, non-unified methods of statistical processing, varying degrees of eutectic purity, etc. The basis of the data estimation technique was the modified least square method, which made it possible to take into account the errors of the experimental data involved. The paper gives the error values of the proposed correlations and the temperature ranges of their applicability. The paper was prepared based on the results of the work of the Thermodynamic Data Center (TDC INPE NRNU MEPhI) of Rosatom State Corporation.

Keywords

Pb-Bi eutectic, heat capacity, thermal conductivity, local speed of sound, regression, density, viscosity

Introduction

The eutectic lead-bismuth alloy has relatively recently begun to be used as a coolant mainly in transportable nuclear power plants (TNPP) (Toshinsky et al. 1999; Zrodnikov et al. 2004). Currently, this eutectic is considered as a possible coolant for promising new generation reactor plants (Dzhangobegov et al. 2015).

A generalized systematic analysis of the experimental results related to the transport and thermodynamic properties of Pb-Bi eutectic was performed by P.L. Kirillov and his colleagues (Kirillov 1998). There are foreign publications summarizing the results obtained by various authors (Sobolev 2010; Handbook 2015). However, they did not include the results obtained by Russian scientists, in particular, the team led by S.V. Stankus (Novosibirsk, Siberian Branch of the Russian Academy of Sciences, S.S. Kutateladze Institute) and the team led by B. B. Alchagirov (Nalchik, Kabardino-Balkarian State University). Numerous experiments performed...
by these scientists were based on advanced methods and tools and thoroughly processed experimental results.

The emergence of new data resulted in the need to clarify and correct existing correlations. A calculation analysis was performed for the following parameters: density, dynamic viscosity, specific heat, thermal conductivity, surface tension, specific electrical resistance, and speed of sound in the alloy.

Since 2020, the correlations proposed in this work have been recommended by the Rosatom State Atomic Energy Corporation for calculating the thermodynamic properties of Pb-Bi eutectic.

**General observations**

The thermodynamic and transport properties of the lead-bismuth eutectic alloy were evaluated on the basis of experimental data presented in 39 works. The authors of this work had information about 1103 experimental points. A direct assessment was performed on 1076 points. Table 1 summarizes the data on experimental work and sources based on which the following correlations were obtained.

To determine the comparative accuracy of the calculated properties according to the recommended correlations, calculations were performed using the formulas from the reference book published by the Nuclear Energy Agency (NEA) (Handbook 2015), which was largely based on the materials given in (Sobolev 2010), (Kirillov 1998) and (Bogoslovskaya et al. 2002). The comparison results for each considered thermodynamic or transport property are presented below in Table 2.

**Table 1.** Data on sources and temperature ranges.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of works*</th>
<th>Period, yrs</th>
<th>Temperature range, K</th>
<th>Total number of points</th>
<th>Number of evaluated points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>12</td>
<td>1952–2015</td>
<td>400–1225</td>
<td>594</td>
<td>594</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>8</td>
<td>1954–2015</td>
<td>400–1300</td>
<td>155</td>
<td>137</td>
</tr>
<tr>
<td>Specific heat</td>
<td>3</td>
<td>1954–1973</td>
<td>400–950</td>
<td>19</td>
<td>11</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>9</td>
<td>1923–2008</td>
<td>400–1000</td>
<td>122</td>
<td>121</td>
</tr>
<tr>
<td>Surface tension</td>
<td>10</td>
<td>1951–2008</td>
<td>400–1250</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>Specific electrical resistance</td>
<td>3</td>
<td>1954–2008</td>
<td>400–1050</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Speed of sound</td>
<td>4</td>
<td>1975–2009</td>
<td>400–1400</td>
<td>112</td>
<td>112</td>
</tr>
</tbody>
</table>

* The number of works based on which the analysis was performed

Table 2. Relative errors of regression equations (1)–(7) for the lead-bismuth eutectic parameters in comparison with the data from (Kirillov 1998, Handbook 2015), %.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>This paper</th>
<th>Kirillov (1998)</th>
<th>NEA (Handbook 2015)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0.25</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>3.22</td>
<td>24.0</td>
<td>4.95</td>
</tr>
<tr>
<td>Specific heat</td>
<td>1.83</td>
<td>2.07</td>
<td>37.86</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>5.3</td>
<td>–</td>
<td>3.87</td>
</tr>
<tr>
<td>Surface tension</td>
<td>1.21</td>
<td>1.17</td>
<td>1.4</td>
</tr>
<tr>
<td>Specific electrical resistance</td>
<td>2.43</td>
<td>3.99</td>
<td>2.5</td>
</tr>
<tr>
<td>Local speed of sound</td>
<td>0.4</td>
<td>–</td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Recommended regression correlations and their errors**

1. The regression equation for the density of Pb-Bi eutectic was obtained based on an analysis of 12 experimental works (Lyon 1952; Miller 1952; Kutateladze et al. 1958; Bonilla 1964; Alchagirov et al. 2003a, b; Stankus et al. 2004, 2005a, b, 2006, 2008; Khairulin et al. 2005), which provide information on 594 experimental points obtained in 1952–2015. The data were processed according to the method briefly described in (Chusov et al. 2019).

The recommended correlation for the density of Pb-Bi eutectic is written as

\[
\rho = 1.1 \cdot 10^4 - 1.22 \cdot T, \quad (1)
\]

where \(T\) is the temperature, K; \(\rho\) is the density, kg/m\(^3\). The applicability range is 400–1225 K. A graphic presentation of the experimental material with a regression curve is shown in Fig. 1. The temperature-density dependences of pure Pb and Bi are also plotted here. All the experimental data are rather densely placed on the approximation line. As expected, the experimental points for the eutectic were between the dependences for pure lead and bismuth. The marks S1–S4 on the legend near the names of some authors mean that the data are given on several series of experiments.

2. The equation for calculating the dynamic viscosity of Pb-Bi eutectic in the liquid phase was obtained based on the analysis of 12 experimental works (Lyon 1952; Miller 1952; Kutateladze et al. 1958; Kirillov 1998; Kirillov 2008; Plevachuk et al. 2008; Filippov et al. 2016) and is written as

\[
\mu = 8.65 \times 10^4 + 1.77 \times 10^5 \exp(-T/200.5), \quad (2)
\]

where \(\mu\) is the dynamic viscosity, Pa·s. Relation (2) is applicable in the temperature range of 400–1300 K.

The calculation results for relation (2) are graphically presented in Fig. 2. In total, in (Kirillov 1998, Miller 1952, Kutateladze et al. 1958, Plevachuk et al. 2008, Kirillov 2008, Filippov et al. 2016), information was given...
on 155 experimental points, of which 137 were involved in the assessment.

3. The correlation for the specific heat \( c_p \) of Pb-Bi eutectic in the liquid phase was obtained on the basis of the processed data from (Miller 1952, Kutateladze et al. 1958, Hultgren et al. 1973):

\[
c_p = 147.0 \text{ J/(kg\times K)}. \tag{3}
\]

The applicability range is 400–950 K. The experimental data are shown in Fig. 3. In the three cited papers, only 19 experimental points are indicated. To justify relation (3), 11 experimental points were used.

4. The recommended correlation for calculating the thermal conductivity of Pb-Bi eutectic in the liquid phase, obtained on the basis of the experimental data from (Dzhanagobegov et al. 2015; Kirillov 1998; Holman 1964; Brown 1923; Powell and Tye 1958; Chirkin 1968; Pokrovsky et al. 1969; Kazakova et al. 1984; Novakovic et al. 2002; Pastor Torres 2003; Semenchenko 1957; Stremousov and Solomin 1975; Kazys et al. 2006), is written as

\[
\lambda = 3.615 + 0.0172 T - 0.405 \cdot 10^{-5} T^2, \tag{4}
\]

where \( \lambda \) is the thermal conductivity, W/(m\times K). The correlation is applicable in the range of 400–1000 K. As a whole, 122 experimental points were obtained in the above studies. The calculated estimation was performed on 121 experimental points.

The graph for equation (4) is shown in Fig. 4.

5. The regression equation for the surface tension coefficient of lead-bismuth eutectic was obtained on the basis of the experimental data from (Miller 1952; Plevachuk et al. 2008; Kirillov 2008; Gromov et al. 1993; Chirkin 1968; Pokrovsky et al. 1969; Kazakova et al. 1984; Novakovic et al. 2002; Pastor Torres 2003; Semenchenko 1957) and is written as

\[
\sigma = (441.1 - 0.0711 \cdot T) \cdot 10^{-3}, \tag{5}
\]

It should be noted that the experimental data presented in this figure diverge on average by 25%. This spread is apparently caused by a high content of impurities in bismuth. Unfortunately, in the majority of the experimental studies, there is no information on the percentage of impurities in the initial experimental material. The authors of this work could only assume their presence. Based on this, the fundamental arguments in selecting data for the assessment were the facts of the deviation of the trend of the experimental data presented in a particular work from the general trend and the explicit “loss” of individual points from the general data array.
where $\sigma$ is the surface tension coefficient, N/m. The temperature range is 400–1370 K.

The graph for equation (5) and experimental data are shown in Fig. 5.

6. The regression equation for the electrical resistance of lead-bismuth eutectic in the liquid phase was obtained based on an analysis of the experimental data from (Miller 1952; Kirillov 2008; Plevachuk et al. 2008) and is written as

$$r = (88.71 + 0.052 \cdot T) \times 10^{-8},$$

where $r$ is the specific electrical resistance, Ohm$\times$m. The temperature range is 400–1050 K.

The graph for equation (6) and experimental data are shown in Fig. 6. The approximation was performed at 24 experimental points.

7. The local speed of sound in Pb-Bi eutectic was assessed based on an analysis of 112 experimental points from (Stremousov and Solomin 1975; Kazys et al. 2006; Hirabayashi et al. 2005; GSSSD 236-2009 2009; Popel' et al. 2010). The regression equation for its calculation is written as

$$u = 1,855 \cdot 10^3 - 0,257 \cdot T,$$

where $u$ is the sound of speed, m/s. The temperature range is 400–1400 K.

The calculation results for relation (7) are presented in Fig. 7.

The values of relative errors for relations (1)–(7) and the errors given in (Kirillov 1998; Handbook 2015) are presented in Tab. 2.

Conclusion

The paper presents semiempirical dependences for calculating several transport and thermodynamic properties of Pb-Bi eutectic in the liquid phase. These dependences were obtained on the basis of an analysis of the experimental data given in 39 works published over the period from 1923 to 2015. The authors indicate the temperature ranges of applicability of the recommended correlations and calculation errors for the seven main parameters characterizing Pb-Bi eutectic, namely: density, dynamic viscosity, specific heat, thermal conductivity, surface tension, specific electric resistance and local sound velocity.

References


