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Research Article

Analytical version of the resonance coupled-channel model for D + T → ^{5}He** → α + n reaction and its application for the description of low-energy D-T and D-^{3}He scattering*

Alexander I. Godes[1], Anna S. Kudriavtseva[2], Vladimir L. Shablov[1]

1 *Obninsk Institute for Nuclear Power Engineering, NRNU MEPhI, 1 Studgorodok, Obninsk, Kaluga reg., 240040 Russian Federation*
2 *National Research Nuclear University MEPhI, 31 Kashirskoe shosse, Moscow, 115409 Russian Federation*

Corresponding author: *Alexander I. Godes* (spartakalex46@mail.ru)

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Abstract

The purpose of the present paper is the formulation of the analytical version of the resonance coupled-channel model (RCCM) originally developed for D + T → ^{5}He** → α + n nuclear fusion reaction. The integral in the denominator of the Breit-Wigner type is examined in the expression for S-matrix elements of binary processes in this model. Imaginary part of this integral determines the energy-dependent decay width for the near-threshold channel. It is demonstrated that this integral can be calculated explicitly with the Binet representation for the ψ-function (the logarithmic derivation of the gamma function). As the result the explicit expression for the S-matrix elements in the form of analytical functions of the channel momenta are obtained and the equivalence of the RCCM and the effective range approximation (Landau – Smorodinsky – Bethe approximation) is established on this basis. This allows expressing the parameters of the RCCM through the model independent system characteristics: the complex scattering length and the complex effective range. Several sets of model parameters of both approaches that provide a good description of the measured data on D + T → α + n reaction and D-T elastic scattering are derived. By this means we find the location of the S – matrix poles on different Riemann sheets which corresponds to $J^\pi = (3/2)^+$ state of ^{5}He and ^{5}Li nuclei. In particular, the location of the resonance (*R*) and shadow (*S*) poles is determined:

^{5}He**: $Z_R = 46.9 - i37.2$ (keV) $Z_S = 81.7 - i3.5$ (keV)

^{5}Li**: $Z_R = 205.7 - i146.8$ (keV) $Z_S = 264.4 + i112.0$ (keV).

Our results agree well with previous findings. The possible generalizations of the results obtained are discussed.

Keywords

Thermonuclear reactions, resonance coupled-channel model, effective range approximation, S-matrix poles, resonance and shadow poles, $J^\pi = (3/2)^+$ state of ^{5}He and ^{5}Li nuclei.

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Introduction

Reactions containing in the final state non-stable quantum systems along with other particles serve as an important source of information about characteristics of non-stable quantum systems. This is of special importance in the cases when the system under examination is difficult or impossible to observe in binary collisions, for instance, neutron excess nuclei ^{9}He, ^{10}He, ρ^0, η^0-mesons, etc. However, a different type of problem emerges in this case, associated with the fact that the process of formation and decay of non-stable system occurs under the effects of Coulomb and nuclear fields produced by accompanying reaction products. This results in the deviation of observed parameters of the resonance from their values in the case of its isolated excitation and decay. In particular, change of shape of the resonance curve, its half-width, shift of resonance maximum position and change in the correlation between the resonance decay branches along different channels are possible with the latter constituting distinguishing feature of near-threshold resonances (Komarov et al. 1987, 1992, 1996, Fazio et al. 1996, Nemets et al. 2007, Mikhailov et al. 2014, Pavlenko et al. 2010). It was demonstrated in these studies that the above-mentioned effects are mainly associated with effects of Coulomb field of accompanying particles on the resonance decay process.

Near-threshold resonances (Fazio et al. 1996, Komarov et al. 1987), the description of which is usually made with application of the so-called approximation with energy-dependent width (Komarov et al. 1996, Wildermuth and Tang 1999, Nikitiu 1983), are of particular interest. In such case many-body scattering **S**-matrix either has $2l+1$ poles on the complex momentum plane (l being the orbital angular momentum of the resonance) when the resonance corresponds to the pair of fragments one of which is neutral (with exception of the case of s-wave when the number of poles is equal to two), or it has the infinite number of poles when a pair of charged fragments forms the resonance. In both cases one among these poles corresponds to the resonance, the second one corresponds to the so-called "shadow" pole, and it is the presence of the latter pole, which may lead to the observed physical effects (Betan et al. 2018, Miaroshi 1980). Anomalous broadening of the resonance peak corresponding to the second excited state of ^{5}He nucleus as compared with the value equal to 70 keV when this resonance is observed in elastic (n+α) collision (Arena et al. 1989), was revealed. Authors of the cited study suggested that the discovered effect is explained by the effects of the shadow pole.

Resonance coupled-channel model and its connection with effective range approximation

Let us address the issue of determination of parameters of the resonance and shadow poles corresponding to the second exited states of ^{5}He** and ^{5}Li** nuclei based on the resonance coupled-channel model for D + T → α + n reaction (Bogdanova et al. 1991). Element of scattering **S**-matrix S_{11} corresponding to elastic D-T scattering has the following form within the framework of the resonance coupled-channel model (Bogdanova et al. 1991):

$$S_{11}(E) = e^{2i\sigma_0(E)}\left(1 - \frac{i\Gamma_1(E)}{E - E_0 - \gamma I(E) + i(\Gamma_1(E) + \Gamma_2)/2}\right), \quad (1)$$

where $\sigma_0(E)$ is the s-wave Coulomb scattering phase; Γ_2 is the α-n channel decay width; $\Gamma_1(E)$ is the energy-dependent width of decay along the D-T channel: $\Gamma_1(E) = -2\gamma \mathrm{Im}\, I(E)$. The function $I(E)$ is preset in the following form:

$$I(E) = \int_0^\infty \frac{k}{(k^2+\beta^2)^2(K^2-k^2)}(e^{2\pi\eta(k)}-1)^{-1}dk, \quad (2)$$

where $K^2 = 2\mu(E + i0)/\hbar^2$; μ is the reduced mass of the D-T system; $\eta(k) = e^2\mu/(\hbar^2 k) = (ka_c)^{-1}$ is the Coulomb parameter of this system; $a_c = \hbar^2/(e^2\mu)$ is the Bohr radius for the D-T pair. Values E_0, Γ_2, β, γ in (1), (2) are the model parameters. It transpires from (2) that the width

$$\Gamma_1(E) = \pi\gamma(e^{2\pi\eta(K)} - 1)^{-1}/(K^2 + \beta^2)^2 = \gamma K C_0^2(K)/[2(K^2 + \beta^2)^2], \quad (3)$$

where $C_0^2(K) = 2\pi\eta(K)(e^{2\pi\eta(K)} - 1)^{-1}$ is the Gamow multiplier, demonstrates correct threshold behavior (Wildermuth and Tang 1999, Nikitiu 1983). Cross-section of D + T → α + n reaction is equal to:

$$\sigma_{DT\to\alpha n}(E) = 2\pi\cdot(3k_{DT}^2)^{-1}\cdot|S_{12}(E)|^2 = 2\pi\cdot(3k_{DT}^2)^{-1}\cdot(1 - |S_{11}(E)|^2) \quad (4)$$

Or, taking (1) into consideration

$$\sigma_{DT\to\alpha n}(E) = 2\pi\Gamma_1(E)\Gamma_2\cdot/\{3k_{DT}^2[(E-E_0-\gamma \mathrm{Re}\, I(E))^2 + (\Gamma_1(E) + \Gamma_2)^2/4]\}, \quad (4')$$

with $k_{DT} = (2\mu E)^{1/2}/\hbar$. It is possible to calculate integral $I(E)$ represented by (2) explicitly using the Binet formula for ψ-function (logarithmic derivative of gamma-function) (Bateman and Erdelyi 1953)

$$\psi(z) = \frac{d\ln\Gamma(z)}{dz} = \ln z - \frac{1}{2z} - 2\int_0^\infty dt \frac{t}{t^2+z^2}(e^{2\pi t}-1)^{-1}dt, \quad \mathrm{Re}(z) > 0, (5)$$

which will subsequently allow directly implementing analytical extension of **S**-matrix elements on non-physical sheet without referring to the contour deformation method as it was done in (Bogdanova et al. 1991).

For calculating $I(E)$ we transform denominators in (2)

$$[(k^2+\beta^2)^2(K^2-k^2)]^{-1} = (K^2+\beta^2)^{-1}(k^2+\beta^2)^{-1}[(K^2-k^2)^{-1} + (k^2+\beta^2)^{-1}]$$

introducing the variable $x = (ka_c)^{-1}$ and taking (5) into account we find

$$I(E + i0) = (K^2 + \beta^2)^{-1}(A_1 + A_2(E)), \quad (6)$$

where

$$A_1 = [-(a_c\beta)^3/8 - (a_c\beta)^2/4 + a_c\beta\psi'(1/(a_c\beta))/4]/(a_c^2\beta^4),$$
$$A_2(E) = (K^2 + \beta^2)^{-1}[A_3(E) - A_4], \quad (7)$$

$$A_3(E) = -0.5\psi(i/(Ka_c)) + i\pi/4 + iKa_c/4 - 0.5\ln(Ka_c),$$
$$A_4 = -0.5\psi(1/(a_c\beta)) - 0.5\ln(a_c\beta) - a_c\beta/4.$$

As a consequence the expression $E - E_0 - \gamma I(E) + i\Gamma_2/2$ can be written in the following form

$$\gamma\omega(K)/[4(K^2 + \beta^2)^2], \quad (8)$$

where function

$$\omega(K) = 2\psi(i/(Ka_c)) - iKa_c - 2\ln(i/(Ka_c)) - (4E_0\beta^4/\gamma + 4A_1\beta^2 - 4A_4 - i2\Gamma_2\beta^4/\gamma) - (-2\hbar^2\beta^4/(\mu\gamma) + 8E_0\beta^2/\gamma - i4\Gamma_2\beta^2/\gamma + 4A_1) K^2 - (-4\hbar^2\beta^2/(\mu\gamma) + 4E_0/\gamma - i2\Gamma_2/\gamma)K^4 + 2\hbar^2K^6/(\mu\gamma). \quad (9)$$

Thus, poles of scattering **S**-matrix on non-physical sheet of wave numbers are zeros of analytical function $\omega(K)$ where $K = k_1 - ik_2$, $\arg K < 0$.

The obtained result allows establishing connection between the resonance coupled-channel model and the effective range approximation for the system of charged particles in the presence of absorption (Karnakov et al. 1990, 1991). The following expression is written for the element of **S**-matrix $S_{11}(E)$ in effective range approximation instead of (1):

$$S_{11}(E) = e^{2i\sigma 0(E)}(\mathrm{ctg}\,\delta_0(K) + i)/(\mathrm{ctg}\,\delta_0(K) - i). \quad (10)$$

Nuclear-Coulomb shift δ_0 is given using the following expression (Landau and Lifshitz 1977):

$$D(K)\cdot\mathrm{ctg}\,\delta_0(K) + 2h(K) = -a_c/a_0 + 0.5r_0a_cK^2, \quad (11)$$

where a_0 is the scattering length; r_0 is the effective range; $D(K) = 2\pi/[\exp(2\pi/(Ka_c)) - 1]$ is the Coulomb barrier penetrability; $h(K) = \mathrm{Re}\psi(i/(Ka_c)) + \ln(Ka_c)$.

Formula (11) is the Landau – Smorodinsky – Bethe approximation (Landau and Lifshitz 1977), and, in the presence of absorption, the scattering length and the effective range become complex values (Karnakov et al. 1990, 1991). Since $D(K) = \pi\mathrm{cth}(\pi/(Ka_c)) - \pi$ and $\mathrm{Im}\psi(ix) = 0.5/x + \pi/2\ \mathrm{cth}\ \pi x$ than the following equation is valid

$$\omega_1(K) = iD - D\mathrm{ctg}\,\delta_0(K) = 2\psi(i/(Ka_c)) + i/(Ka_c) - 2\ln(i/(Ka_c)) - \varphi(K^2) \quad (12)$$

with

$$\varphi(K^2) = -a_c/a_0 + 0.5r_0a_cK^2 = \alpha_0 + \alpha_1a_c^2K^2 - i(\beta_0 + i\beta_1a_c^2K^2), \quad (12')$$

where β_0 and β_1 parameters are not negative (Karnakov et al. 1990, 1991). We obtain from (4'), (12) and (12')

$$\sigma_{DT\to\alpha n}(E) = 8\pi(\beta_0 + \beta_1a_c^2K^2)D(K)/[3k_{DT}^2|\omega_1(K)|^2],\ K = k_{DT}. \quad (13)$$

Comparison of functions $\omega(K)$ and $\omega_1(K)$ demonstrates that if terms proportional to K^4 and K^6 are neglected in (9) then the connection can be established between the parameters of the models under discussion in the following form

$$\beta_0 = 2\Gamma_2\beta^4/\gamma, \quad a_c^2\beta_1 = 4\Gamma_2\beta^2/\gamma,$$
$$\alpha_0 = 4E_0\beta^4/\gamma + 4A_1\beta^2 - 4A_4, \quad (14)$$
$$a_c^2\alpha_1 = 8E_0\beta^2/\gamma - 2\hbar^2\beta^4/(\mu\gamma) + 4A_1.$$

Taking into account the first two expressions in (14) equation (4') for D + T → α + n reaction cross-section is reduced in the approximation specified above to the following form:

$$\sigma_{DT}(E) = 8\pi(\beta_0 + \beta_1a_c^2K^2 + \beta_0K^4/\beta^4)D(K)/[3k_{DT}^2|\omega_1(K)|^2], \quad (15)$$

which coincides with (13) within the accuracy of the term proportional to K^4.

Determination of model parameters and positioning of S-matrix poles of D + T → α + n and D + ^{3}He → α + p processes in the neighborhood of resonance energies of ^{5}He(3/2)$^+$ and ^{5}Li(3/2)$^+$ nuclei

Two sets of parameters of the resonance coupled-channel model well matching the cross-section of the thermonuclear fusion D + T → α + n reaction are presented in (Bogdanova et al. 1991) for the case of ^{5}He(3/2)$^+$ nucleus with one of the presented sets of parameters in good agreement with measured data on the cross-section of elastic D-T scattering. Expressions (14) lead to the following two sets of effective range approximation.

1. Resonance coupled-channel model (Set 1)

E_0 = 2.263 MeV; Γ_2 = 0.3738 MeV; β = 0.3399 fm^{-1}; γ = 0.00703 MeV·fm^{-4}.

Effective range approximation

α_0 = 0.3003; α_1 = 0.0798; β_0 = 0.1422; β_1 = 0.0043.

2. Resonance coupled-channel model (Set 2)

E_0 = 3.686 МэВ; Γ_2 = 0.5479 МэВ; β = 0.3399 Фм$^{-1}$; γ = 0.1105 МэВ·Фм$^{-4}$.

Effective range approximation

α_0 = 0.2894; α_1 = 0.0878; β_0 = 0.1328; β_1 = 0.004.

Let us note that E_0 and Γ_2 parameters are taken from (Bogdanova et al. 1991), β parameter corresponds to the value of в E_f: $\beta = (2\mu E_f)^{1/2}/\hbar$ which is similar for both options and is equal to 2 MeV, and parameter γ is recalculated according to the values of cognominal variable presented in (Bogdanova et al. 1991).

The following set of parameters of the effective range approximation was obtained in (Karnakov et al. 1990) on the basis of analysis of thermonuclear fusion reaction:

$$\alpha_0 = 0.233;\ \alpha_1 = 0.121;\ \beta_0 = 0.0785;\ \beta_1 = 0.00798.$$

The following third set of parameters of the resonance coupled-channel model is obtained as the result:

$E_0 = 0.7853$ MeV; $\Gamma_2 = 0.1741$ MeV; $\beta = 0.1845$ fm^{-1}; $\gamma = 0.0051$ MeV·fm^{-4}.

All sets of parameters presented above were additionally analyzed to verify agreement with experimental data using parametrization of experimental data on the D + T → α + n reaction cross-section (Bosch and Hale 1992) more recent as compared with (Bogdanova et al. 1991, Karnakov et al. 1990, 1991) (Fig.1). As it was originally expected, all six sets of parameters are in good agreement with the parametrization in question. The above fact confirms the conclusion drawn before in (Bogdanova et al. 1991) that experimental data of only one type are not sufficient for the determination of true parameters of the models. Because of this reason, the data on elastic D-T-scattering were analyzed similarly to (Bogdanova et al. 1991). Energy dependence of the ratio of elastic D-T-scattering cross-section to the Rutherford cross-section for scattering angle $\Theta = \pi/2$ (Balashko 1965) is presented in Figure 2 in the center-of-mass system:

$$\xi(E) = \frac{d\sigma_{el}(\theta)}{d\Omega}\Big/\frac{d\sigma_R(\theta)}{d\Omega} = \frac{1}{3} + \frac{2}{3}\left|f_c(q) + (S_{11} - S^c_{11})/(2ik_{DT})\right|\Big/\left|f_c(q)\right|^2, \quad (16)$$

where $f_c(q) = -2\mu e^2(\hbar q)^{-2}\exp(2i\sigma_0(E) - i\eta\ln(q^2/(4k_{DT}^2)))$ is the Coulomb scattering amplitude; $\hbar q$ is the transmitted momentum; S_{11} is the **S**-matrix of elastic D-T-scattering in s-wave; $S^c_{11} = \exp(2i\sigma_0(E))$ is the Coulomb **S**-matrix in s-wave. For angle $\Theta = \pi/2$ expression (16) acquires the following form (Bogdanova et al. 1991):

$$\xi(E) = 1/3 + (2/3)\left|\exp(i\eta\ln 2) - (i/(2\eta))(1 - S_{11}\exp(-2i\sigma_0(E)))\right|^2. \quad (17)$$

The presented values of parameters of the resonanse coupled-channel model and approximation of effective range were applied for determining the poles of scattering **S**-matrix on different sheets of Riemann complex momenta surfaces, i.e. for $K = k_1 - ik_2$, $\arg K < 0$. The property of resonance denominators $\omega(K, \Gamma_2)^* = \omega(-K^*, -\Gamma_2)$ which is easily established on the basis of explicit form of $\omega(K)$ function (9) was applied here. Calculated values of parameter of the resonance and shadow poles of scattering **S**-matrix for D-T system in the neighborhood of energy of state $J^\pi = (3/2)^+$ of ^{5}He nucleus are presented in Table 1.

Let us note that the models, parameters of which describe well the behavior of nuclear fusion reaction (not only those presented above), produce close values of resonance parameters while the parameters of the shadow pole can significantly differ. It was established on the basis of comparison of different sets of parameters with experimental data that the set of parameters from (Bogdanova et al. 1991) equivalent to Set 1, is the best. Results presented in (Bogdanova et al. 1991) recently found confirmation in (Betan et al. 2018): $Z_S \approx 82 - i3.5$ (keV). Position of Coulomb poles was determined beside that in agreement with results in (Karnakov et al. 1990).

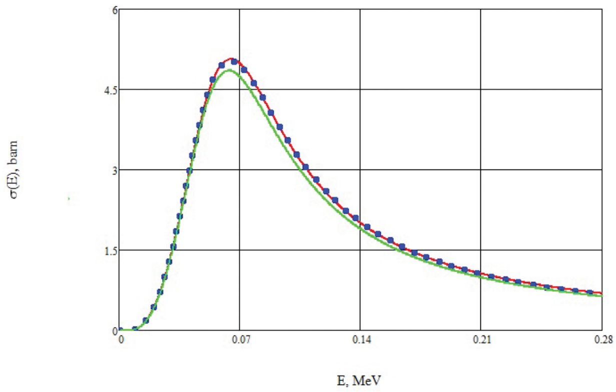

Figure 1. The energy dependence of nuclear fusion D + T → α + n reaction cross-section: solid line is the parametrization in (Bosch and Hale 1992); "dotted" line is the resonance coupled-channel model (Set 1); dashed line is the effective range approximation.

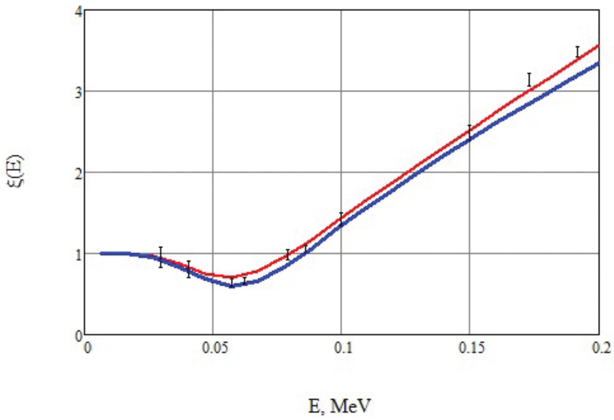

Figure 2. Comparison of the ratio of elastic D-T-scattering to the Rutherford cross-section with experimental data: solid line is the resonance coupled-channel model (Set 1); "dotted line" is the effective range approximation.

Table 1. Poles of amplitude of low-energy D-T-scattering.

Parameter set	Resonance pole	Shadow pole
Resonance coupled-channel model (Set 1)	$K_R = (1.334 - i0.465)/a_c$ $Z_R = 46.9 - i37.2$ keV	$K_S = (-1.652 + i0.034)/a_c$ $Z_S = 81.7 - i3.5$ keV
Resonance coupled-channel model (Set 2)	$K_R = (1.334 - i0.468)/a_c$ $Z_R = 46.8 - i37.5$ keV	$K_S = (-1.649 + i0.003)/a_c$ $Z_S = 81.7 - i0.3$ keV
Effective range approximation (Karnakov et al. 1990)	$K_R = (1.33 - i0.45)/a_c$ $Z_R = 46 - i36$ keV	$K_S = (-1.61 - i0.15)/a_c$ $Z_S = 77 + i14$ keV

The method developed was applied to the description of $D + {}^3He \rightarrow \alpha + p$ fusion reaction. It was established in this case that the best set of parameters for this reaction from the viewpoint of agreement with parametrization (Bosch and Hale 1992) is the set calculated on the basis of effective range approximation (Karnakov et al. 1991):

$$\alpha_0 = 0.1627;\ \alpha_1 = 0.1555;\ \beta_0 = 0.01631;\ \beta_1 = 0.00927.$$

Parameters of the resonance coupled-channel model were determined on the basis of expressions (14):

$$E_0 = 0.642\ \text{MeV};\ \Gamma_2 = 0.0835\ \text{MeV};\ \beta = 0.154\ \text{fm}^{-1};$$
$$\gamma = 0.00792\ \text{Mev}\cdot\text{fm}^{-4}.$$

This results in the following parameters for resonance and shadow poles:

$$K_R = (1.383 - i0.443)/a_c,\ Z_R = 205.7 - i146.8\ \text{keV},$$
$$K_S = (1.538 - i0.3)/a_c,\ Z_S = 264.4 + i1112.0\ \text{keV},$$

which agrees with results in (Karnakov et al. 1991).

Conclusion

Four-parameter representation was obtained for **S**-matrix elements in the resonance coupled-channel model (Bogdanova et al. 1991) allowing investigating analytical structure of **S**-matrix in the neighborhood of near-threshold energy on the basis of explicit expressions derived for it in the form of analytical channel momenta functions. An important consequence of the obtained expressions is the establishment of equivalence of the resonance coupled-channel model and the effective range approximation for the system of charged particles with absorption (Karnakov et al. 1990, 1991). As the result, parameters of the model can be expressed through model-free characteristics of the system – complex scattering length and complex effective range. The obtained results are planned to be used in the future for the description of near-threshold resonances for ^{7}Li, ^{8}Be nuclei and for investigating decay characteristics of these nuclei, as well as of ^{5}He and ^{5}Li in multi-particle nuclear reactions.

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