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Research Article

Circuit design solutions for the reactimeters*

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Abstract

A number of issues pertaining to comparative analysis of possible options for algorithmic and circuit embodiment of reactimeters were examined from the standpoint of the general theory of measuring instruments and the theory of digital filters. Structural diagrams of the linear part of the reactimeter, as well as the functional algorithms and their numerical implementation are described in terms of transient characteristics and transfer functions. Parallel, straight, canonical, symmetrized, lattice and ladder block structural diagrams are examined. The corresponding difference equations are given. The obtained results allow comparing possible circuit design solutions from the viewpoint of a number of criteria: the complexity of the elemental composition (the number of integrators, summation units, multipliers, delay elements), the number of necessary computing operations, the identifiability of the hardware function of the reactimeter, the coherence between the calculated and the measured values, the sensitivity to parameter uncertainties, etc. The possibility of considering the equations of the reactimeter as autoregressive is demonstrated, which ensures adaptability of the reactimeter under operating conditions. Certain algorithms for identification of the transient response characteristic and transfer function of the reactimeter are indicated. The possibility is shown of using identical algorithms in the main computing unit for solving the direct and inverse problems of nuclear reactor kinetics for ensuring consistency between the calculated and the measured reactivity values. Upper and lower estimated reactivity values are suggested for the moment of switching on the reactimeter. Implementation of such estimations in the reactimeter design allows minimizing the time needed for reaching by the reactimeter of the operating mode. Certain methodological simplifications were used in the development of ladder and lattice circuit design solutions. The database containing parameters of the instrumental functions of the circuit design solutions of the reactimeters is available on a public website. A number of tasks and directions for further research are identified.

Keywords

Reactimeter, instrumental function, circuit design solution, variant analysis

Introduction

The purpose of the present study is the examination of certain issues pertaining to the comparative analysis of options of algorithmic and circuit design implementation of reactimeters from the viewpoint of criteria of the general theory of measurement instruments (Woschni 1964) and digital signal processing (Sergienko 2003). Hardware (transfer) function of a measuring instrument can be represented in a number of different structural forms. These forms determine the circuit and algorithmic design solutions for the instrument and predetermine the complexity of the element composition of the circuits, the measurement accuracy, the tempo of getting the next measurement,

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the possibility of identification of the hardware function, quality of noise suppression, sensitivity to parameter uncertainties, time for reaching the operational mode, etc. However, description and comparison of possible circuit design solutions for reactimeters taking into account the above criteria have not been implemented so far.

Practically all reactimeter designs (see, in particular, (Yuferov 2003, Lititsky et al. 1982, Ostapenko and Shostak 1966, Voronin and Ostapenko 1968, Vyushin and Volegov 1969, Punch and Schwiegger 1975, Kim 1977, Peterson and Larson 1978, Sarylov et al. 1979, Aleksakov et al. 1985, Technical description 1987, CVR-10 Reacimeter, Polozov and Sikorin 1987, Grachev et al. 1986, Aksenov et al. 1990, Volkov et al. 1999, Technical characteristics 2010, Lititsky and Zhukov 2013)) are based on the converted solution of a set of differential equations of nuclear reactor kinetics taking into account six groups of delayed neutrons. Here, the hardware function of the linear part of the reactimeter is represented as the sum of exponents which results in the parallel block structural diagram (six parallel integrating RC circuit in the analogue reactimeter design). Group parameters of delayed neutrons predictably changing in the process of fuel irradiation campaign of the nuclear reactor (NR) serve as the coefficients and power exponents. Therefore, the task of reactimeter adaptation to the conditions of specific NR, i.e. the justification of adequacy of the indicated parameters and their refinement by calculation or experiment, is always present. In practical terms this task is solved on the basis of indirect data supported by calculation estimation of fuel burnup (Voronin et al. 1985, Sivokon and Poznyakov 1990, CVR-11 Reacimeter, Fadeev and Moiseev 2005). This allows the transition to one of preliminarily prepared sets of parameters of delayed neutrons. On the other hand, the algorithms are known for direct estimation of the hardware function according to the recorded transients or noises (Yuferov and Ibragimov 2005, Yuferov 2005). At the same time, such experimental hardware function must be represented as the sum of exponents for adaptation of reactimeters in conventional configuration. This requires complex iteration calculations (Keepin 1965) while not guaranteeing sufficient accuracy (Lanczosh 1961). This is the reason why alternative reactimeter circuit design solutions with non-exponential representation of the hardware function ensuring the possibility of its prompt identification in the NR operational modes are of interest. It is preferable as well if these solutions:

– Minimized the number of structural elements and operations
– Applied simple reactivity assessment algorithms correlated with reactivity calculation procedures accepted in the codes simulating the NR dynamics (Recommendations 2011, Yuferov 2017);
– Ensured short time for reaching by the reactimeter the intended operation mode.

Certain aspects pertaining to the above issues are addressed in the present paper. The main attention is paid to the circuit design solutions for the reactimeter as the digital filter. Corresponding analogue circuits can be constructed following well known relations (Sergienko 2003).

Reactimeter autoregressive equations

The set of differential equations of NR kinetics is reduced to the following integral equation (Yuferov 2017):

$$v(t) = n(t)r(t) - \int_0^t h(t-\tau)v(\tau)d\tau + Q(t). \tag{1}$$

Integration of the equation by parts reduces the latter to the form containing only power readings:

$$n(t) = \rho^*(t)n(t) + \int_0^t g(t-\tau)n(\tau)d\tau, \quad g(\tau) = (dh(\tau)/d\tau)\Lambda/\beta_{ef}. \tag{2}$$

Reactimeter equations straightforwardly follow from (1), (2). Thus, following Eq. (1),

$$r(t) = \alpha(t) + [\int_0^t h(t-\tau)dn(\tau) - Q(t)]/n(t), \tag{3}$$

where $n(t)$ is the nuclear reactor power level; $v(t) = dn/dt$ is the rate of variation of reactor power; $\alpha(t)$ is the inverse reactor period; $r(t)$ is the reactivity according to the Λ-scale: $r = \rho/\Lambda = 1/\Lambda - 1/l$; Λ is the generation time; l is the prompt neutron lifetime; ρ is the absolute reactivity; ρ^* is the reactivity expressed in units of effective fraction of delayed neutrons β_{ef}: $\rho^* = \rho/\beta_{ef}$.

Reactivity according to Λ-scale is the relative rate of reproduction of prompt neutrons, i.e. the algebraic sum of relative rates of generation (probability $1/\Lambda$) and loss (probability $1/l$) of prompt neutrons. Theoretical shape of the kernel of the following delayed neutron integral (DNI)

$$Y(t) = \int_0^t h(t-\tau)dn(\tau)$$

has the following form:

$$h(t-\tau) = \frac{\beta_{ef}}{\Lambda}\sum_{j=1}^{J} d_j \exp(-\lambda_j(t-\tau)), \tag{4}$$

where d_j, λ_j are the group parameters of delayed neutrons; β_{ef}/Λ is the probability of generation of delayed neutrons (fraction of neutrons "spent" for the generation of precursors of delayed neutrons). DNI kernel has the meaning of the function of reproduction of precursors and describes the reactimeter hardware function in the problem under examination. Identical dimensionality of the values r, α and β_{ef}/Λ simplifies the comparison of relative rates of processes in the NR, analysis of "period-reactivity" dependences (Yuferov 2009) and the task of selection of reactivity measurement units (Koshelev and Kolesov 1992).

Equations (1) – (3) ensure the required accordance between the solutions of the direct (calculation of reactor power dynamics) and the inverse (calculation or measurement of reactivity) problems if these problems use identical algorithms in their discrete realization

$$v_k = r_k n_k - Y_k \beta_{ef}/\Lambda + Q_k \tag{5}$$

In the case of constant values of the source and the reactivity (1), (2) and (5) can be considered as the autoregression equations relative to the constants r, Q, ρ^* and sampling values of hardware functions h, g. If sufficient number of measurements of the rate or reactor power is available the above equations are efficiently solved relative to the indicated regression factors using appropriate variant of the least square method (Yuferov 2012, Marple 1990). Values of hardware functions h, g obtained using this approach are used in quadrature formulas for calculating the delayed neutron integral (Yuferov 2017) without the need of its reversal to the form (4). However, the nature of decay of precursors of delayed neutrons predetermines the interval of attenuation of h and g functions equal to approximately 300 and 80 seconds, respectively. This will require extremely large number of multiplier and delay units in the hardware implementation. Therefore, it is expedient to examine other circuit design solutions starting from the theoretical shape of the DNI kernel (4) determining the possible structures of the reactimeter transfer functions.

Reactimeter transfer functions

In order to analyze algorithmic and circuit design options of the reactimeter let us write down its equation as the convolution equation by separating the linear part and converting it to the structure of linear filter

$$v(t) + \int_0^t h(t-\tau)v(\tau)d\tau = g(t) \tag{6}$$

with input signal $v(t)$ and response $g(t) = r(t)n(t) + Q(t)$. The general form of the transfer function (TF) for this equation has the form $W(s) = 1 + (\beta_{ef}/\Lambda)H(s)$. However, we can ignore the unit bearing in mind the typical relations between the reactivity $r(t)$ and the inverted period $\alpha(t)$ in Equation (3). This is in correspondence with widely accepted practice of calculation of reactivity and design of reactimeters. It is appropriate to introduce the probability of generation of delayed neutrons $h_0 = \beta_{ef}/\Lambda$ explicitly as the amplification coefficient for the linear part of the reactimeter. This value can be the result of independent calculation or experimental estimation and it provides in this capacity additional possibility for testing adequacy of reactimeter adaptation. Thus, for comparing the circuit design solutions of the reactimeter it is sufficient to examine the structural transformations of the transfer function of the delayed neutron integral $H(s)$ normalized to h_0.

The direct discretization of the delayed neutron integral in the case when certain quadrature formula is applied in the calculations without accounting for the exponential representation of the kernel (4),

$$Y_k = T\sum_{l=0}^{m}(A_{k,k-l}h_l)v_{k-l},$$

is characterized by the transfer function written in the following *non-recursive structural form*

$$H(z) = T\sum_{l=0}^{m} c_l z^{-l},$$

where $c_l = A_{k,k-l}h_l$, $A_{k,k-l}$ are the coefficients in the applied quadrature formula. Here, the number of operations in one step increases in the process of calculations until the number m (the number of readings required for accounting for the interval of attenuation of the hardware function) is reached. This number depends on the selected quadrature formula and determines the number of multiplier units and delay units in the hardware implementation of the digital reactimeter with the given form of the TF.

Analogue transfer function in *the following parallel structural form*:

$$H(s) = \sum_{j=1}^{J}\frac{d_j}{s+\lambda_j}. \tag{7}$$

corresponds to the standard exponential form of the DNI kernel (4).

Conventional circuit design solutions for analogue reactimeters are based on this form. Evident transformations of the $H(s)$ TF produce the *straight-line*

$$H(s) = A(s)/B(s) = \sum_{j=0}^{J-1}a_j s^j \Big/ \sum_{j=0}^{J} b_j s^j, \quad b_J \equiv 1 \tag{8}$$

and the *cascade*

$$H(s) = \prod_{j=1}^{J-1}(s-\zeta_j) \Big/ \prod_{j=1}^{J}(s+\lambda_j) \tag{9}$$

structural forms.

In particular, the parameters of form (8) are equal to:

$$b_k = \sum_{i_{k+1}=1}^{k+1}\sum_{i_{k+2}=i_{k+1}+1}^{k+2}\cdots\sum_{i_J=i_{J-1}+1}^{J}\left(\prod_{j=k+1}^{J}\lambda_{i_j}\right), \quad k=\overline{0,J-1};$$

$$a_k = \sum_{i_{k+2}=1}^{k+2}\sum_{i_{k+3}=i_{k+2}+1}^{k+3}\cdots\sum_{i_J=i_{J-1}+1}^{J}\left[\left(\prod_{j=k+2}^{J}\lambda_{i_j}\right)\left(1-\sum_{j=k+2}^{J}d_{i_j}\right)\right], \quad k=\overline{0,J-1}.$$

Sensitivity of these parameters to the delayed neutron constants is described by the following relations:

$$\frac{\partial b_k}{\partial \lambda_l} = \sum_{i_{k+1}=1}^{k+1} \sum_{i_{k+2}=i_{k+1}+1}^{k+2} \cdots \sum_{i_J=i_{J-1}+1}^{J} \left(\frac{\partial}{\partial \lambda_l} \prod_{j=k+1}^{J} \lambda_{i_j} \right), \qquad k = \overline{0, J-1};$$

$$\frac{\partial a_k}{\partial \lambda_l} = \sum_{i_{k+2}=1}^{k+2} \sum_{i_{k+3}=i_{k+2}+1}^{k+3} \cdots \sum_{i_J=i_{J-1}+1}^{J} \left[\frac{\partial}{\partial \lambda_l} \left(\prod_{j=k+2}^{J} \lambda_{i_j} \right) \left(\beta_{\text{ef}} - \sum_{j=k+2}^{J} \beta_{i_j} \right) \right], \qquad k = \overline{0, J-1};$$

$$\frac{\partial a_k}{\partial \beta_l} = \sum_{i_{k+2}=1}^{k+2} \sum_{i_{k+3}=i_{k+2}+1}^{k+3} \cdots \sum_{i_J=i_{J-1}+1}^{J} \left[\frac{\partial}{\partial \beta_l} \left(\beta_{\text{ef}} - \sum_{j=k+2}^{J} \beta_{i_j} \right) \left(\prod_{j=k+2}^{J} \lambda_{i_j} \right) \right], \ \beta_l = d_l \beta_{\text{ef}}, \ k = \overline{0, J-1}.$$

Complexity of element compositions of the hardware implementation of the reactimeter is characterized by the number of parameters and operators s (or z) in the transfer functions. The number of parameters determines the number of multiplier units in the circuit design implementation, and the number of operators determines the number of integrating elements (for the analogue implementation) or delay units (for the digital implementation). These characteristics determine as well the number of arithmetic operations in respective codes for reactivity calculations.

Discrete analogue of the parallel structural form (7) follows from the condition of coincidence of transfer characteristics of the analogue and discrete implementations of the reactimeter linear part:

$$H(z) = Tz \sum_{j=1}^{J} d_j / (z + z_j), \quad z_j = -\exp(-\lambda_j T), \tag{10}$$

where T is the discretization step. Discrete analogues of transfer functions (8) and (9) are constructed on the basis of this TF. The codes for calculating parameters of the indicated TF and corresponding coefficients of sensitivity to variations of constants of delayed neutrons are presented in (Yuferov 2007). The following set of difference equations for estimation of the interval of delayed neutrons in Equation (5) corresponds to the transfer function (10):

$$x^j{}_k = Td_j v_k - z_j x^j{}_{k-1}, \quad x^j{}_{-1} = 0, \quad j = \overline{1, J}, \quad Y_k = \sum_{j=1}^{J} x^j{}_k.$$

Different options of such single-step discretization (quadrature formula) are used in the equation of digital reactimeter – the inverted solution of the kinetics equation (Yuferov 2003, Lititsky et al. 1982). Their advantage is the possibility of parallelization of calculations and fixed number of operations in each step. The latter is ensured by the separation of variables in kernel (4) which is predetermined by the exponential structure of the kernel.

Discrete analogue of the straight-line TF (8) can be written as the product $H(z)=T[B(z)]^{-1}A(z)$, where

$$A(z) = \sum_{j=0}^{J-1} \mu_j z^{-j}, \qquad B(z) = 1 + \sum_{j=1}^{J} \gamma_j z^{-j}, \tag{11}$$

and the input signal is initially processed by the block $A(z)$. In such case the DNI estimation is as follows:

$$Y_k = T \sum_{j=0}^{J-1} \mu_j v_{k-j} - \sum_{j=1}^{J} \gamma_j Y_{k-j}. \tag{12}$$

Replacing the DNI readouts in (12) with variables in Equation (5) we obtain the equations for the intensity of generation of prompt neutrons $u(t) = r(t)n(t)$:

$$u_k = \sum_{j=0}^{J} \eta_j v_{k-j} - \sum_{j=0}^{J} \gamma_j u_{k-j} - q_k. \tag{13}$$

This allows decreasing the number of operations in the calculations of reactivity $r_k = u_k/n_k$.

Change of the order of operations for processing the input signal by permutation of blocks $A(z)$ and $B(z)$ produces the *canonic structural form* to which the difference equations

$$x_k = v_k - \sum_{j=1}^{J} \gamma_j x_{k-j}, \qquad Y_k = T \sum_{j=0}^{J-1} \mu_j x_{k-j}. \tag{14}$$

correspond.

In the given case only the input signal x_k is memorized which reduces the number of delay elements by two times. Similar result is obtained in pairwise grouping of summation terms with the same index in Equations (12) or (13). For Equation (12) we obtain:

$$Y_k = \sum_{j=0}^{J} (T\mu_j v_{k-j} - \gamma_j Y_{k-j}), \quad \gamma_0 = 0, \quad \mu_J = 0.$$

Here, each calculation block (expression in the brackets) uses separate integrator but, however, the potential gain is associated with the fact that these blocks can operate in parallel. It is appropriate to call such layout *symmetrized* since the input and output values passing through common delay elements similarly processed in the main calculation block.

Cascade structural forms are implemented when at least one of the TF polynomials is factorized. When only linear multipliers (10) are used in the denominator $B(z)$ of the discrete TF (11) the cascade form is described by the following equations:

$$x_k = T \sum^{J-1} \mu_j v_{k-j}, \quad y_k{}^0 = x_k, \quad y_k{}^j = y_k{}^{j-1} - z_j y_{k-1}, \quad j \quad 1, J,$$

or

$$y_k{}^0 = v_k, \quad y_k{}^j = y_k{}^{j-1} - z_j y_{k-1}{}^j, \quad j = \overline{1, J}, \quad Y_k = T \sum_{j=0}^{J-1} \mu_j y_{k-j}{}^J,$$

It is appropriate to call such form *the cascade form by the output*. Similarly, *the cascade form by the input* is obtained by the factorization of the numerator $A(z)$ with retaining the denominator $B(z)$ in the form (11).

Cascade structural forms are of interest since standard bilinear or biquadratic blocks can be used in the hardware implementation. However, in this case group parameters of delayed neutrons must be known.

Lattice-like structural form is the version of cascade implementation not requiring the knowledge of zeros and poles of transfer functions. For obtaining the lattice-like structure all-pass filter, i.e. filter with transfer function $C(z)/B(z)$ numerator of which $C(z)$ contains mirror permutation of coefficients of the polynomial $B(z)$: $c_j \rightarrow b_{J-j}$, is constructed on the basis of denominator $B(z)$ of the straight-line TF (11).

From the characteristic property of the all-pass filter

$$\frac{C^j(z)}{B^j(z)} = \left(q^j + z^{-1}\frac{C^{j-1}(z)}{B^{j-1}(z)}\right) \Bigg/ \left(1 - z^{-1}q^j\frac{C^{j-1}(z)}{B^{j-1}(z)}\right), \quad j = \overline{J,1} \qquad (15)$$

follow the equations of constraint for the lattice cascades

$$\begin{pmatrix} C^j \\ B^j \end{pmatrix} = \begin{pmatrix} z^{-1} & q^j \\ z^{-1}q^j & 1 \end{pmatrix} \begin{pmatrix} C^{j-1} \\ B^{j-1} \end{pmatrix}$$

and the algorithm for calculating the cascade coefficients q^j:

$q^j = b_j^{\,j}$, $C^j(z) = z^{-j}B^j(z \rightarrow 1/z)$, т.е. $c_j^{\,j} = b_{J-j}^{\,j}$,
$B^{j-1}(z) = (B^j(z) - q^jC^j(z))/(1 - q^jq^j)$, $j = J, J-1, ..., 1$.

Here the superscript index is the number of the cascade and the subscript index is the number of the coefficient in the polynomial.

According to the characteristic property (15) inputs of adjacent cascades are coupled as follows: $x^{j-1}(z) = [B^{j-1}(z)/B^j(z)]\, x^{j-1}(z)$. Therefore, input of the j-th cascade is coupled with the input signal of the reactimeter $v(z) = x^J(z)$ by the relation $x^j(z) = [B^j(z)/B(z)]v(z)$. It follows herefrom that output of the j-th cascade (equal to $y^j(z) \equiv [C^j(z)/B^j(z)]\, x^j(z)$) is coupled with input signal of the reactimeter $v(z)$ as $y^j(z) = [C^j(z)/B(z)]\, v(z)$. The latter relation means that the circuit design equivalent to the TF (11) can be implemented by summing outputs $y^j(z)$ with appropriate weights p_j and the following DNI estimation is obtained:

$$Y_k = \sum_{j=1}^{J} p_j y_k^{\,j}.$$

Weights p_j are found from the representation of the TF (11) numerator as the following sum

$$A(z) = \sum_{j=1}^{J} p_j C^j(z)$$

by solving the set of linear equations linking the coefficients of polynomials $A(z)$ and $C^j(z)$ with corresponding exponential factors. The above described lattice-like structure ensures stability of the solution and weak sensitivity to the uncertainties of coefficients (Marple 1990).

Ladder structural form also refers to the cascade type. By implementing standard transformations based on the interpretation of relations (8) and (11) in terms of transfer functions for two-poles and quadripoles it allows reducing the description of the hardware function of the reactimeter to three parameters. Ladder block structural diagram can be obtained by interpretation of the transfer function (8) as the impedance $H = E/I$ of a certain two-pole in which separate segments are calculated in steps. In the first step the two-pole is considered as two successive branches with resistance Z_1 and conductivity Y^* so that $E = HI = IZ_1 + I/Y^*$. In the second step the branch with conductivity Y^* is represented with parallel branches with resistances Z_2 and Z^* so that $HI = IZ_1 + I/(1/Z_1 + 1/Z^*)$. By these means one section of the ladder structure is formed. After this, the two operations are repeated in relation to the following branch with resistance equal to Z^* and so on. Such method of building up the structure corresponds to the procedure for calculating Z_i coefficients by expanding the TF (8) into the continued fraction:

$H = E/I = A/B = [Z_1(Z_2 + Z^*) + Z_2Z^*]/(Z_2 + Z^*) = Z_1 + 1/(1/Z_2 + 1/Z^*)$.

Such constructions are not unequivocal. Different options of equivalent structures are possible resulting in the decrease of the number of segments in case of correct selection of resistances Z_i. In particular, n-section ladder structure obtained after the completion of the expansion into the continued fraction using the above described algorithm can be regarded as a loaded cascade of quadripoles connected in series each of which consists of the resistance Z_i and resistance Z_{2i} connected in parallel to the load. Such structure is described by the transfer matrix calculated as the product of transfer matrices of the quadripoles:

$$\mathbf{T} = \prod_{1}^{n} \begin{pmatrix} 1 + Z_i / Z_{2i} & Z_i \\ 1/Z_{2i} & 1 \end{pmatrix}.$$

Elements of this matrix t_{ij} allow calculating the parameters of equivalent quadripole. In particular, parameters of equivalent Π-structure are equal to $Z_1 = t_{12}/(t_{22} - 1)$, $Z_2 = t_{12}$, $Z_3 = t_{12}/(t_{11} - 1)$.

Following the interpretation of the transfer function H accepted in the construction of the ladder structure, current I at the cascade input acts as the input signal while the output signal is the voltage E at the cascade input. Therefore, in case of Π-structure we obtain $H = E/I = Z_1(Z_2 + Z_3)/(Z_1 + Z_2 + Z_3)$.

Similar procedures are applicable to discrete TF of the straight-line structural form numerator and denominator of which are represented by polynomials (11). For them the first step of expansion of the TF into the continued fraction produces $H(z) = 1/(c_1z^{-1} + 1/H_1(z))$, where $H_1(z) = A(z)/Q_1(z)$, $Q_1(z)$ is the remainder of division of polynomials $B(z)/A(z)$, c_1 is the real coefficient. In such case the delayed neutron integral $Y_k = H_1(z)(Tv_k - c_1Y_{k-1})$. This transformation separates within the structure the negative feedback segment – the summand c_1Y_{k-1}. Since the power of polynomial $A(z)$ is higher than the power of polynomial $Q_1(z)$ we obtain in the second step the expansion $H_1(z) = c_2 + H_2(z)$ separating within the structure parallel branch with transfer coefficient c_2. Following this we repeat the first step in relation to the transfer function $H_2(z)$, etc.

Identification of hardware function of the reactimeter

Parameters of the above described circuit design solutions can be identified according to experimental data which solves the problem of reactimeter adaptation. Here the reduction of the number of elements of the circuit structure is possible if it is discovered that the identification produces zero values for respective parameters.

Identification of hardware function of the reactimeter is implemented in the most straightforward way (from the viewpoint of calculation) for the recursive structural form

$$H(z) = T\sum_{l=0}^{m} c_l z^{-l}$$

in the situation when the leaving by the reactor of the steady-state operational mode is provoked by prompt pulsed or stepwise disturbance of the reactivity or the source. In such case the expression for direct estimation of the hardware function follows from Equation (5):

$$h_k = -[v_k + \sum_{l=1}^{k-1}(A_{k,k-l}v_{k-l})\cdot h_l]/(A_{k,0}v_0), \quad k = 1,2,... \quad .$$

As applicable for the method of instantaneously removed source this formula takes the form

$$h_k = n_k / n_0 - v_k / v_0 - \sum_{l=1}^{k-1}(v_{k-l}/v_0)\cdot h_l, \quad k = 1,2,... \quad ,$$

if the DNI is calculated using the method of rectangles and measurements are performed with time step of one second. Examples of such identification are provided in (Yuferov 2005). Drawbacks of this methodology are predetermined by the possibility of its application only for special modes of functioning of the NR and the necessity of setting up a series of dedicated identification experiments.

Results of noise identification of the non-recursive reactimeter hardware function using Burg's method (Marple 1990) are described in (Yuferov 2012). Using the Burg's technique, the cascade coefficients of the ladder structural form mentioned above are calculated as well and the applied adequacy criterion allows obtaining the optimal number of cascades without recalculation of previously obtained coefficients. The methodology in question appears to be the most attractive one since it can be applied for obtaining real time estimations in steady-state conditions of reactor operation, as well as ensures direct identification of two types of circuit design solutions.

Parameters of the straight-line structural form (11) can be found using Pade approximations (Vinogradov et al. 1987) by implementing the identification of the non-recursive form:

$$\sum_{l=0}^{m} c_l z^{-l} = (1 + \sum_{j=1}^{J}\gamma_j z^{-j}) / \sum_{j=0}^{J}\mu_j z^{-j}.$$

Account of initial conditions at the time of turning on the reactimeter

Equation (1) was obtained in the assumption that the reactor was operated before the time moment t = 0 in steady-state mode (subcritical or critical): $r(t)n(t) + Q(t) = 0$, $Y(t) = 0$ for $t < 0$. In such case the initial condition for solving the direct problems of reactor kinetics using Equation (1) always has the standard form $v(0) = r(0)n(0) + Q(0)$, where $r(0)$, $Q(0)$ are the initial bursts of reactivity or of the source predetermining exit of the reactor from the steady-state operation mode. General case must be examined in the solution of the inverse problem in the assumption that DNI is not equal to zero at the moment of turning on the reactimeter because the current DNI value is determined by the preceding power behavior during the memory interval of transient characteristics $h(t)$ or $g(t)$.

Estimation of the accumulated DNI can be obtained on the basis of the principle of dynamic similarity of the pre-history, in pursuance with which the preceding behavior of the system and its boundary conditions can be selected arbitrarily within the framework of the accepted model if they result in the observed current conditions. In the problem under study the current state of the reactor is the reactor power and the rate of its evolution at the time moment t of turning on the reactimeter. If, in particular, it is assumed that the reactor was brought to the indicated state from the steady-state operation conditions by exponential growth of reactor power with period equal to the current instantaneous period $p(t) = 1/\alpha(t)$, then the estimation of reactivity at the time moment t is determined by the in-hour equation in pre-asymptotic form:

$$r(t) = \alpha(t)\{1 + (\beta_{\mathrm{ef}}/\Lambda)\sum_{j=1}^{J}(d_j/(\alpha(t)+\lambda_j))[1-\exp(-(\alpha(t)+\lambda_j)t)]\}.$$

Here, the lower estimation of reactivity (according to Λ-scale) equal to $\alpha(t)$ follows from the assumption that the reactimeter is turned on at the moment when the reactor is exiting from steady-state operation mode. If, however, it is assumed that exponential excursion lasted infinitely long ($t = \infty$) prior to the moment of turning on the reactimeter, then the upper estimate is obtained as follows:

$$r(t) = \alpha(t)\cdot[1 + (\beta_{\mathrm{ef}}/\Lambda)\sum_{j=1}^{J} d_j/(\alpha(t)+\lambda_j)] \approx \alpha(t)\cdot(\beta_{\mathrm{ef}}/\Lambda)\cdot T_{\mathrm{del}},$$

where T_{del} is the lifetime of delayed neutrons. This estimate majorizes the estimates corresponding to any other path for transition to the current state. It can only overestimate the real value of reactivity and, therefore, satisfies the nuclear safety requirements.

Approximate expression in the last formula is applicable for any circuit design solutions for presetting the value of reactivity at the time moment of turning on the reactimeter. It corresponds to the standard settings according to the doubling period T_2 > 10 s (in this case the value α in the denominator of the presented formulas can be neglected) and to the characteristic value of the generation time (allowing neglecting the first summand). Availability in the design of digital reactimeter of the possibility to perform such estimations allows reducing to zero the time needed for reaching by the reactimeter of its operation mode.

Conclusion

1. Possible options of circuit design implementation of reactimeter are described in terms of structural forms of transfer functions for the reactimeter linear part. Discrete TF were obtained under the condition of coincidence of transfer characteristics of the analogue and discrete implementations of the reactimeter linear part. Respective difference equations are given the structure of which determines the number of required multiplier units, memory elements and summator units in the hardware implementation of the reactimeter.
2. Represented difference equations can be used both in the calculations of reactivity and for calculating the reactor power dynamics. This unifies the direct and the inverse problems of the NR dynamics and ensures accordance between the measured and calculated reactivity values.
3. Algorithms are described for identification of parameters of transfer functions ensuring reactimeter adaptation in operational conditions. From the viewpoint of simplicity of calculations, the non-recursive structural form appears to be the most attractive.
4. Relations are given that relate the coefficients of various structural forms of TF. Calculation of coefficients of transfer functions was performed. The current array of parameters of the reactimeter transfer functions is posted on public website.
5. Due to the linearity of the main computational block the suggested form of the reactimeter equation does not require the transition to small disturbance equations traditionally applied (Hetrick 1971) in the analysis of reactivity disturbances.

For further work under the considerate subject, it is appropriate to specify the following tasks:

– Construction of transfer functions of the digital reactimeter on the basis of different discretization methods applying, for instance, the bilinear transformation or *z*-shape (Kuo 1980);
– Calculation of parameters for possible circuit design solutions for the known systems of delayed neutron data;
– Comparative analysis of suggested algorithms and circuit design solutions from the viewpoint of quality of noise suppression;
– Derivation of dispersion equations for the reactimeter (Yuferov 2016) as applied to different structural forms of the reactimeter transfer functions;
– Generalization of the presented difference equations for multipoint models of NR dynamics;
– Comparative analysis of the described circuit design solutions as applied to specific inventory of hardware components.

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