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Research Article

Lawson criterion for different scenarios of using D-^{3}He fuel in fusion reactors*

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Abstract

The paper is devoted to refining the Lawson criterion for three scenarios of using D-^{3}He fuel in fusion reactors (fully catalyzed and non-catalysed D-D cycles and a D-^{3}He cycle with ^{3}He self-supply). To this end, a new parameterization of the D + ^{3}He → p + ^{4}He fusion reaction cross-section and astrophysical factor has been developed based on the effective radius approximation (Landau-Smorodinsky-Bethe approximation), which is a model-free theoretical approach to investigating near-threshold nuclear reactions, including resonant reactions. In the framework of this approximation, experimental data from studies in the NACRE II and EXFOR libraries, believed to provide the most reliable results to date, have been described within the accuracy declared in the studies in question in the energy range of 0 to 1000 keV, and the fusion reactivity averaged over the Maxwell distribution has been calculated. The results obtained are in good agreement with the calculations based on the *R*-matrix theory and the NACRE II fusion reactivity data. For the fully catalyzed D-D cycle and the cycle with ^{3}He self-supply, the Lawson criterion and the triple Lawson criterion have been calculated based on solving the equations of the stationary process kinetics in a fusion reactor for three fuel ions (D, ^{3}He, and T) taking into account the potential for external supply of ^{3}He and p and ^{4}He impurity ions removed from the reaction zone. The parameters of the triple Lawson criterion found are as follows: $n\tau T = 6.42 \cdot 10^{16}$ cm^{-3}·s·keV ($T = 54$ keV) for the fully catalyzed D-D cycle, $n\tau T = 1.03 \cdot 10^{17}$ cm^{-3}·s·keV ($T = 45$ keV) for the cycle with ^{3}He self-supply, and $n\tau T = 4.89 \cdot 10^{16}$ cm^{-3}·s·keV ($T = 67$ keV) for the non-catalyzed D-D cycle with equimolar D-^{3}He fuel.

Keywords

thermonuclear reactions, effective radius approximation, cross-section and astrophysical factor of D + ^{3}He → p + ^{4}He fusion reaction, Lawson criterion for different D-^{3}He fuel use scenarios

Introduction

Since the mid-1950s, the purpose of investigations in the field of controlled nuclear fusion (CNF) has been to reach and exceed the Lawson criterion (Lawson 1957; Zhdanov et al. 2017; Wurzel and Hsu 2022) that defines the conditions for achieving a self-sustained nuclear reaction. This requires three conditions as follows to be simultaneously satisfied:

- thermonuclear fuel needs to be heated to such temperatures with which the kinetic energy of nuclei becomes sufficient for their tunneling through the Coulomb barrier with a noticeable probability;
- the concentration of the plasma initiated in the course of heating needs to be rather high;

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- the plasma temperature and concentration need to be maintained for a definite time, τ, referred to as the plasma confinement time.

In the simplest case, when one does not take into account different conversion efficiencies of the energy generated in the fusion reactor (FR), which is used then in part or in full to heat and confine the plasma and is neglected by the radiation losses, the Lawson criterion has the form

$$n_e\tau \geq 12kT/(\langle\sigma v\rangle Q), \qquad (1)$$

where k is the Boltzmann constant, $\langle\sigma v\rangle$ is the fusion reactivity the value of which is assumed for isothermal plasma, that is, for temperature T (it is assumed that there is a dominant reaction as in the event of deuterium-tritium plasma), and Q is the kinetic energy of the charged particles resulting from the fusion reaction. Despite being approximate, formula (1) is of a universal nature (that is, does not practically depend on the thermonuclear facility type) and gives a reference value for the plasma confinement parameter $n_e\tau$, the achievement of which leads to a self-sustained fusion reaction. Besides, formula (1) shows the important role played by the correct definition of quantity $\langle\sigma v\rangle$. A change of even several percent in the reaction rate is capable to affect in a noticeable way the parameters of advanced thermonuclear facilities, specifically, the plasma confinement parameter. It is exactly the Lawson criterion that defines the smallest possible frequency of fusion reactions per second required for sustaining steadily the reaction in a material medium. Although this quantity fits the reaction cross-section averaging based on the Maxwell distribution with temperature T in the situation under consideration (while real processes are not necessarily described by the Maxwell distribution, as, for example, in the event of plasma in an strong magnetic field (Artsimovich 1964), $\langle\sigma v\rangle$ is also the reference value for calculating real fusion systems, this case requiring knowing the reaction reactivity in a broad temperature range. We shall point to astrophysical investigations, another field of science, which requires knowing values $\langle\sigma v\rangle$ with a good accuracy.

The purpose of this study is to determine more precisely the Lawson criterion for fusion reactors on the base of D-^{3}He fuel viewed as the immediate competitor to D-T fuel. The major advantage of a D-^{3}He reactor, as compared with a D-T reactor, is its low level of the neutron flux from plasma with which the lifetime of the reactor first wall is expected to reach 30 to 40 years (Khvesyuk and Chirkov 2000; Ryzhkov and Chirkov 2017). The D-^{3}He fuel cycle is characterized, however, by much higher Lawson criterion values. We shall remind that, as of the late 1950s, the Lawson criterion was $n_e\tau = 10^{14}$ cm^{-3}·s (T = 17 keV) for equimolar D-T fuel, and $n_e\tau = 10^{15}$ cm^{-3}·s (T = 87 keV) for equimolar D-^{3}He fuel (Shirokov and Yudin 1980), as more current estimates are as follows (Basko 2007): $n_e\tau = 1.6\cdot10^{14}$ cm^{-3}·s (T = 26 keV) for D-T fuel; $n_e\tau = 8.1\cdot10^{14}$ cm^{-3}·s (T = 105 keV) for D-^{3}He fuel. The differences that have occurred are associated to a large extent with determining more accurately the temperature dependence of the reaction reactivity. The paper will present the results of calculating the temperature dependence of the D + ^{3}He → p + ^{4}He reaction reactivity based on the parametrization of its astrophysical factor in the framework of the effective radius approximation (Bethe 1949; Barit and Sergeev 1969; Landau and Lifshitz 1977; Karnakov et al. 1991), and a comparison will be provided against three most common parametrizations of value $\langle\sigma v\rangle$. Three options will be considered for the D-^{3}He fuel cycle (Khvesyuk and Chirkov 2000; Stott 2005):

- a fully catalyzed D-D cycle in which the generated tritium and ^{3}He are fully used as secondary fuel;
- a non-catalyzed D-D cycle when intermediate ^{3}He and T are removed after they give up most of their thermonuclear kinetic energy for plasma heating but prior to further combustion with deuterium;
- ^{3}He self-supply mode.

Parametrization of the astrophysical factor and temperature dependence of the D + ^{3}He → p + ^{4}He reaction reactivity in different approaches

Since the 1950s, multiple experimental and theoretical studies have been undertaken on the fusion cross-sections and thermal reactivities (see the overview in Bosch and Hale 1992). Different parametrizations were developed for the reaction cross-sections and reactivities (Kozlov 1962; Fowler et al. 1967; Peres 1979; Caughlan and Fowler 1988; Belov and Kalitkin 2017; Godes et al. 2019), the differences in which are associated both with their methodological basis and the fact that they were created at different times and were based so on varying experimental material. As applied to the D + ^{3}He → p + ^{4}He reaction, the developed parametrizations, which are resonant in the low-energy region, can be divided into two groups:

Group 1

Parametrizations based on physical models and approximations, including:

- the R-matrix Wigner theory that formed the basis for the parametrizations of cross-sections and rates for the key thermonuclear reactions in Bosch and Hale 1992;
- the Breit-Wigner approximation used in Kozlov 1962; Fowler et al. 1967; Caughlan and Fowler 1988;
- a model-free approach – the effective radius approximation (ERA) (Barit and Sergeev 1969; Karnakov et al. 1991);
- the resonance coupled channel model (Godes et al. 2019).

The advantage of these parametrizations is their immediate physical meaning, correct threshold behavior, use of few adjustable parameters, and possibility for being practically employed in other fields of physics, e.g., in nuclear spectroscopy, and the disadvantage is a limited applicability area.

Group 2

Parametrizations based on mathematical methods for approximation of experimental data, including:

- a Padé approximation of the reaction cross-section and rate with the correct threshold behavior (Peres 1979) used further in Bosch and Hale 1992;
- the regularized double period method (Belov and Kalitkin 2017), in which regularization by the A.N. Tikhonov stabilizer is used to exclude nonphysical oscillations of an approximating trigonometric series caused by major errors in experimental points.

Both methods are suitable for solving a specialized problem, that is, to determine the temperature dependence of the reaction reactivities but are unfit for addressing other problems. Besides, the double period method requires a large number of experimental points which is problematic in the event of the reaction of interest since the consideration includes studies with an insufficiently defined methodological framework (see the discussion in (Moller and Besenbaher 1980; Krauss et al. 1987; Bosch and Hale 1992; Geist et al. 1999).

The results of this study will be compared with data from the most common of the current parametrizations (Kozlov 1962; Caughlan and Fowler 1988; Bosch and Hale 1992) and NACRE II (Xua et al. 2013).

The NACRE II (Nuclear Astrophysics Compilation of Reactions) parametrization contains data on the rates of 34 exoergic reactions caused by charged particles with a mass number of $A < 16$. In NACRE II, the reaction rates are presented in a tabulated format in a temperature range of $1 \cdot 10^6$ K $\leq T \leq 1 \cdot 10^{10}$ K and contains experimental data from before 2013. The tables present values of the low, high and adopted estimates for value $N_A\langle\sigma v\rangle$. NACRE II is based on calculations of cross-sections using different theoretical models (the distorted wave Born approximation, the Breit-Wigner approximation, etc.).

We shall note that in the event of the D + ^{3}He → p + ^{4}He reaction, the previous NACRE version uses the results obtained in Fowler et al. 1967, Caughlan and Fowler 1988, based on the Breit-Wigner approximation with a constant width, though it would be more reasonable to employ an approach using an approximation with an energy-dependent width (Wildermuth and Tang 1999) for the near-threshold thermonuclear resonance of the ^{5}Li** nucleus through which this reaction proceeds in the low-energy region. In NACRE II, resonance reactions are described using the distorted waves Born approximation (for the non-resonance part of the amplitude) combined with the Breit-Wigner approximation (for the resonance part of the amplitude) taking into account, where required, the dependence of the resonance width on energy.

Parametrization of the astrophysical factor and temperature dependence on the D + ^{3}He → p + ^{4}He reaction rate based on the effective radius approximation

The effective radius approximation (ERA) is a model-free approach and operates on experimentally observed quantities, including scattering length, effective radius and potential shape parameter.

For low-energy scattering in a system of two charged particles, this approximation is based on the following expression for the **S**-matrix element $S_{11}(E)$ of the s-wave elastic scattering

$$S_{11}(E) = e^{2i\sigma_0(E)} \frac{ctg\delta_0(k)+i}{ctg\delta_0(k)-i}, \tag{2}$$

where $\sigma_0(E) = arg\Gamma(1 + i\eta)$ is the Coulomb scattering phase, and η = η(k)is the Coulomb parameter

$$\eta = \frac{Z_1 Z_2 e^2 m_r}{\hbar^2 k} \tag{3}$$

or $\eta = (ka_c)^{-1}$; a_c is the Bohr radius for a pair of synthesized nuclei with reduced mass m_r:

$$a_c = \hbar^2/(Z_1 Z_2 e^2 m_r). \tag{4}$$

In the framework of ERA, the nuclear – coulomb shift, $\delta_0(k)$, is determined by the expression

$$a_c^{-1}\,[D(k)ctg\delta_0(k) + 2h(k)] = -a_0^{-1} + 0.5r_0k^2, \tag{5}$$

where a_0 is the scattering length, r_0 is the effective radius, and $D(k)$ is the Coulomb barrier penetrability

$$D(k) = 2\pi/(e^{2\pi\eta} - 1), \tag{6}$$

$$h(k) = Re\psi(i\eta) - \ln(\eta) = Re\psi(i/(ka_c)) + \ln(ka_c), \tag{7}$$

where $\psi(z)$ is the logarithmic derivative of the gamma function. In the event of absorption, if any, the nuclear-coulomb shift becomes a complex value, and the scattering length and the effective radius become so complex values as well (Karnakov et al. 1991).

Then the following equality takes place

$$\omega(k) = D(k)ctg\delta_0(k) - iD(k) = \varphi(k^2) - 2h(k) \tag{8}$$

with the function $\varphi(k^2)$ of the form

$$\varphi(k^2) = -a_c/a_0 + 0.5r_0a_ck^2 = a_0 + a_1k^2 - i(\beta_0 + \beta_1a_c^2k^2). \tag{9}$$

In a general case, the terms with k^4 and k^6 may be taken into account in the function $\varphi(k^2)$. As the result, the reaction cross-section is equal to

$$\sigma_r(E) = g(\pi/k^2)(1 - |S_{11}|^2), \tag{10}$$

where g is the spin factor, $(2J + 1)/[(2S_1 + 1)(2S_2 + 1)]$, equal, in the event of the D + ^{3}He → p + ^{4}He reaction, to 2/3, or

$$\sigma_r(E) = 8\pi\beta(k)D(k)/(3k^2|\omega(k)|^2). \quad (11)$$

We shall present the expression (11) as

$$\sigma_r(E) = \frac{8\pi}{3k^2}\frac{\beta(k)D(k)}{(\alpha(k)-2h(k))^2+(\beta(k)+D(k))^2}, \quad (12)$$

$$\alpha(k) = \alpha_0 + \alpha_1(ka_c)^2 + \alpha_2(ka_c)^4,\ \beta(k) = \beta_0 + \beta_1(ka_c)^2 + \beta_2(ka_c)^4.$$

From (12), the following expression is obtained for the astrophysical factor $S(E)=Ee^{2\pi\eta}\sigma_r(E)$:

$$S(E) = \frac{8\pi^2\hbar^2}{3m_r(1-e^{-2\pi\eta})}\frac{\beta(k)}{(\alpha(k)-2h(k))^2+(\beta(k)+D(k))^2}, \quad (13)$$

or (taking into account numerical factors)

$$S(E) = \frac{9.11}{(1-e^{-2\pi\eta})}\frac{\beta(k)}{(\alpha(k)-2h(k))^2+(\beta(k)+D(k))^2}(MeV \cdot b). \quad (14)$$

ERA applicability condition: parameter $kR \le 1$ where R is the radius of action for nuclear forces (Landau and Lifshitz 1977), approximately equal, in accordance with Xua et al. 2013, to 1 Fm, so, with $E \le 1$ MeV, the said parameter does not exceed 0.25.

Besides, ERA is not applicable with $E \le 0.02$ keV when the laboratory electron screening effects manifest themselves in the nuclear reaction cross-sections (Xua et al. 2013).

The ERA parameters, using which the astrophysical factor in Fig. 1 has been calculated and which ensure the approximation of the currently most reliable experimental data from Moller and Besenbaher 1980, Krauss et al. 1987, Geist et al. 1999 with the accuracy stated therein, are as follows:

$$\alpha_0 = 0.117002,\ \alpha_1 = 0.191855,\ \alpha_2 = -0.01225,$$
$$\beta_0 = 0.00937,\ \beta_1 = 0.006658,\ \beta_2 = 0.000582. \quad (15)$$

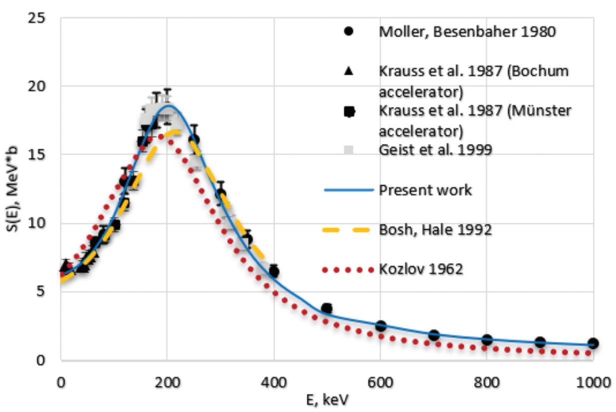

Figure 1. The astrophysical S-factor of the D + ^{3}He → p + ^{4}He reaction.

The presented parameters also agree with the data on the elastic D-^{3}He scattering Balashko 1965 (Fig. 2).

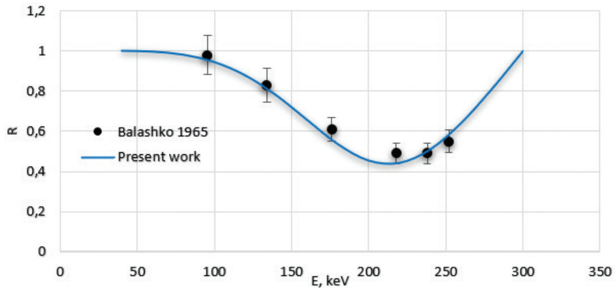

Figure 2. The ratio of the elastic D-^{3}He scattering cross-section to the Rutherford cross-section at a scattering angle of 90 °C in the mass center system.

Another set of parameters based on experimental data in Moller and Besenbaher 1980, Krauss et al. 1987 was obtained in Alper et al. 2021:

$$\alpha_0 = 0.05431,\ \alpha_1 = 0.25077,\ \alpha_2 = -0.02825,$$
$$\beta_0 = 0.00205,\ \beta_1 = 0.00707,\ \beta_2 = 0.00169.$$

The elastic scattering description in this case is somewhat worse and is not provided herein.

The fusion reaction rate is determined as:

$$\langle\sigma v\rangle = \int_0^\infty v\sigma(E)F(E)dE, \quad (16)$$

Where $F(E)$ is the function of the Maxwell distribution by energy:

$$F(E) = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\sqrt{E}e^{-E/(kT)}, \quad (17)$$

The results of calculating the temperature dependence of the rate of the reaction under investigation are shown in Fig. 3.

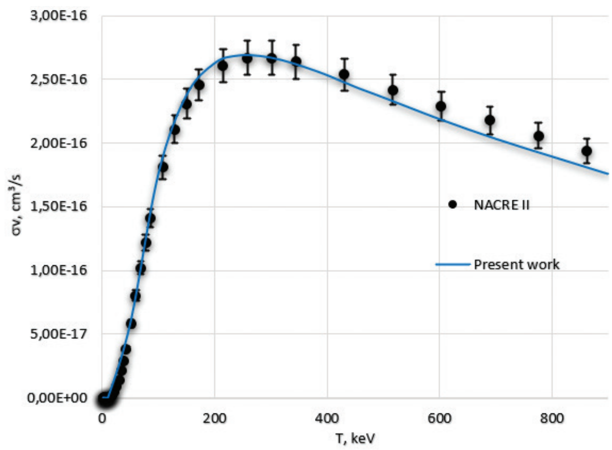

Figure 3. The temperature dependence of the rate of the reaction.

Lawson criterion for different scenarios of using D-^{3}He fuel in fusion reactors

In the currently most common form, the Lawson criterion for fusion reactors with magnetic plasma confinement is written as follows Wurzel and Hsu 2022, Basko 2007, Stott 2005:

$$n\tau = 1.5(1 + \langle Z\rangle)T/[(Q^{-1} + f_c)A_f - A_{br} - A_{e\text{-}e}], \quad (18)$$

where T is the plasma temperature, keV, n is the concentration of ions, Q is the energy gain factor, e.g. the ratio of the volume-average fusion energy to the volume-average energy delivered to the plasma from external sources, f_c is the part of the thermonuclear energy absorbed in plasma, A_f is the function that defines the fusion power, A_{br} and $A_{e\text{-}e}$ are the volume-average power losses via bremsstrahlung radiation on ions and on electrons respectively, $\langle Z\rangle=\sum Z_j n_j/\sum n_j$ is the average charge of plasma ions, and $n_e=\langle Z\rangle n$ is the concentration of electrons. A_f and $f_c A_f$ are defined in the expression (18) by the relations

$$A_f n^2 = 18533\langle\sigma v\rangle_{DHe\to p\alpha} n_D n_{He} + 0.5(3269\langle\sigma v\rangle_{DD\to nHe} + 4033\langle\sigma v\rangle_{DD\to pT})n_D^2 + 17589\langle\sigma v\rangle_{DT\to n\alpha} n_D n_T,$$
$$(1 - f_c)A_f n^2 = 1225\langle\sigma v\rangle_{DD\to nHe}\, n_D^2 + 14028\langle\sigma v\rangle_{DT\to n\alpha} n_D n_T, \quad (19)$$

such that $(1 - f_c)A_f n^2$ is the power comes out via the neutrons.

The neutron energy of 14.028 MeV from the DT → nα reaction was obtained using the alpha particle mass of 4.001506 a.m.u. (Tiesinga et al. 2021) and with regard for the relativistic effects (see the Appendix 1). Functions A_{br} and $A_{e\text{-}e}$ were taken from Khvesyuk and Chirkov 2000. There is no term in (18) that fits the synchrotron radiation – such radiation believed to be fully absorbed in plasma.

We shall consider the fully catalyzed D-D cycle when the T and ^{3}He formed remain in plasma and burn together with deuterium.

The steady-state operation kinetics of a quasi-infinite FR is defined by the condition that there are constant concentrations of D, ^{3}He and T fuel ions and p and ^{4}He impurity ions maintained in plasma. The plasma is assumed to be isothermal, and helium ions are assumed to be doubly ionized. Taking into account the main values in terms of energy production and reaction rates in the temperature region of interest of 50 to 150 keV, the kinetics equations for the D, ^{3}He and T fuel ions and the protons and alpha particles to be removed from the reaction zone are written as follows:

$$\frac{dn_D}{dt} = -\langle\sigma v\rangle_{DHe\to p\alpha} n_D n_{He} - 0.5(\langle\sigma v\rangle_{DD\to pT} + \langle\sigma v\rangle_{DD\to nHe})n_D^2 - \langle\sigma v\rangle_{DT\to n\alpha} n_D n_T + \frac{dn_{D\_ext}}{dt} \quad (20)$$

$$\frac{dn_{He}}{dt} = -\langle\sigma v\rangle_{DHe\to p\alpha} n_D n_{He} + 0.5\langle\sigma v\rangle_{DD\to nHe} n_D^2 + \frac{dn_{He\_ext}}{dt} \quad (21)$$

$$\frac{dn_T}{dt} = 0.5\langle\sigma v\rangle_{DD\to pT} n_D^2 - \langle\sigma v\rangle_{DT\to n\alpha} n_D n_T - \lambda n_T \quad (22)$$

$$\frac{dn_\alpha}{dt} = \langle\sigma v\rangle_{DHe\to p\alpha} n_D n_{He} + \langle\sigma v\rangle_{DT\to n\alpha} n_D n_T - \Gamma_\alpha \quad (23)$$

$$\frac{dn_p}{dt} = \langle\sigma v\rangle_{DHe\to p\alpha} n_D n_{He} + 0.5\langle\sigma v\rangle_{DD\to pT} n_D^2 - \Gamma_p. \quad (24)$$

dn_{D_ext}/dt describes the FR supply with deuterium, and dn_{He_ext}/dt describes the FR supply with ^{3}He; λ in (22) is the rate of the ^{3}H beta decay; Γ_α and Γ_p describe the removal of alpha particles and protons from the FR: $\Gamma_j = n_j/\tau_j$, (τ_j is the confinement time for particles of type j). The presented form of the kinetics equations means, specifically, that the hydrogen and carbon cycle reactions, the T + T, T + ^{3}He and ^{3}He + ^{3}He reactions, and the energy of the tritium beta decay are neglected. Besides, the presented system of equations does not take into account the escape of fuel ions from the reaction zone as it was done, e.g., in Stott 2005. Since $dn_T/dt = 0$ the equilibrium concentration of tritium is equal to

$$n_T = \frac{0.5\langle\sigma v\rangle_{DD\to pT}}{\langle\sigma v\rangle_{DT\to n\alpha} n_D + \lambda} n_D. \quad (25)$$

Formula (25) allows estimating the contribution from the tritium beta decay for which λn_T and $\langle\sigma v\rangle_{DT\to n\alpha} n_D n_T$ need to be compared. We shall take into account that $\lambda \approx 2.9\cdot 1\square^9$ s$\square^1$ and in a temperature range of $1\cdot 10^8$ to $1\cdot 10^9$ K:

$$\langle\sigma v\rangle_{DT\to n\alpha} n_D \sim (10^{-18} \div 2\cdot 10^{-15}) n_D \text{cm}^3\cdot\text{s}^{-1} \quad (26)$$

so with $n_D \geq 1\cdot 10^{12}$cм$^{-3}$, the contribution from beta decay in (25) may be neglected. Then

$$n_T = \frac{0.5\langle\sigma v\rangle_{DD\to pT}}{\langle\sigma v\rangle_{DT\to n\alpha}} n_D = \alpha\gamma_T n, \gamma_T = \frac{0.5\langle\sigma v\rangle_{DD\to pT}}{\langle\sigma v\rangle_{DT\to n\alpha}}, \quad (27)$$

where $n = n_D + n_T + n_{He} + n_p + n_\alpha$ is the concentration of plasma ions, and α is the relative concentration of deuterium ($n_D = \alpha n$). Expression (27) coincides with the findings in Wurzel and Hsu 2022, Khvesyuk and Chirkov 2022. Guided by the NACRE II data with $T = 105$ keV (the anticipated temperature of the Lawson criterion for the fully catalyzed mode), one can estimate the tritium generation:

$N_A\langle\sigma v\rangle_{DD\to pT} = 1.54\cdot 10^7$ cm^{3}mol$^{-1}s^{-1}$, $N_A\langle\sigma v\rangle_{DT\to n\alpha} = 4.76\cdot 10^8$ cm^{3}mol$^{-1}s^{-1}$ so $n_T/n_D = 0.5\langle\sigma v\rangle_{DD\to pT}\, n_D/(\langle\sigma v ñ_{DT\to n\alpha} n_D + \lambda) \approx 1.6\cdot 10^{-2}$.

Similarly, the equilibrium concentration of ^{3}He is equal (in the ^{3}He self-supply mode) to:

$$n_{He} = \frac{0.5\langle\sigma v\rangle_{DD\to nHe}}{\langle\sigma v\rangle_{DHe\to p\alpha}} n_D = \alpha\gamma_{He} n, \;\; \gamma_{He} = \frac{0.5\langle\sigma v\rangle_{DD\to nHe}}{\langle\sigma v\rangle_{DHe\to p\alpha}} \quad (28)$$

In the event of supply with ^{3}He, if any, expression (28) shall be substituted for

$$n_{He} = \frac{0.5\langle\sigma v\rangle_{DD\to nHe}}{\langle\sigma v\rangle_{DHe\to p\alpha}} n_D + \frac{dn_{\text{He ext}}/dt}{\langle\sigma v\rangle_{DHe\to p\alpha} n_D} = \alpha\gamma_{He} n\delta, \quad (29)$$

where δ > 1 in the supply mode is the external parameter that can be attributed to the volume average power of the FR.

The volume-average power of the fusion reaction $P_f = A_f n^2$ in the new notation is described as:

$$P_f = 18533\langle\sigma v\rangle_{DHe\to p\alpha}\alpha^2\gamma_{He} n^2 + 0.5\cdot 4033\langle\sigma v\rangle_{DD\to pT}\alpha^2 n^2 + 0.5\cdot 3269\langle\sigma v\rangle_{DD\to nHe}\alpha^2 n^2 + +17589\langle\sigma v\rangle_{DD\to n\alpha}\alpha^2\gamma_T n^2. \quad (30)$$

The volume-average power, P_c, generated in plasma by charged particles is

$$P_f = 18533\langle\sigma v\rangle_{DHe\to p\alpha}\alpha^2\gamma_{He}n^2 + 0.5\cdot 4033\langle\sigma v\rangle_{DD\to pT}\alpha^2 n^2 + 0.5\cdot 819\langle\sigma v\rangle_{DD\to nHe}\alpha^2 n^2 ++ 3561\langle\sigma v\rangle_{DT\to n\alpha}\alpha^2\gamma_T n^2 = A_c n^2 = f_c A_f n^2. \quad (31)$$

In the ^{3}He supply mode, parameter γ_{He} in (30) and (31) needs to be multiplied by δ.

The following ratios take place:

$$(n_D + n_{He} + n_T + n_P + n_\alpha)/n = \alpha + \alpha\gamma_{He}\delta + \alpha\gamma_T + n_P/n + n_\alpha/n = 1. \quad (32)$$

It stems from (22) – (24) that $\Gamma_\alpha = \Gamma_p$ in the steady-state mode, so

$$\frac{n_P}{n_\alpha} = \frac{\tau_P}{\tau_\alpha} = \xi = \frac{\langle\sigma v\rangle_{DHe\to p\alpha}\gamma_{He}\delta + 0.5\langle\sigma v\rangle_{DD\to pT}}{\langle\sigma v\rangle_{DHe\to p\alpha}\gamma_{He}\delta + \langle\sigma v\rangle_{DT\to n\alpha}\gamma_T}. \quad (33)$$

The results of determining the Lawson criterion and the triple Lawson criterion $n\tau T$ for the thermonuclear reaction ignition mode, ($Q = \infty$), and for the case with $\xi = 1$ are presented below.

^{3}He self-supply mode ($\delta = 1$)

It was found as the result of a numerical simulation that parameter $n\tau T$ in the self-supply mode is determined in the deuterium concentration range of $0.56 \le \alpha \le 0.9$. The minimum for this parameter (triple Lawson criterion) fits value $\alpha = 0.89$ and is equal to:

$$n\tau T = 1.03\cdot 10^{17}\ \text{cm}^{-3}\cdot\text{s}\cdot\text{keV}\ (T = 45\ \text{keV}).$$

With value α being as that, the Lawson criterion looks as follows:

$$n\tau = 1.48\cdot 10^{15}\ \text{cm}^{-3}\cdot\text{s}\ (T = 130\ \text{keV}).$$

The presented results is qualitatively consistent with the results obtained in Khvesyuk and Chirkov 2022 for the triple Lawson criterion $n\tau T = (1\text{–}2)\cdot 10^{17}\ \text{cm}^{-3}\cdot\text{s}\cdot\text{keV}$ ($T = 50\text{–}70$ keV).

The concentration of plasma ions and its charge characteristics are as follows:

$$n_D = 0.89n,\ n_{He} = 0.094n,\ n_T = 0.0046n,$$
$$n_\alpha = 0.0055n,\ n_p = 0.0055n,$$
$$\langle Z\rangle = 1.1,\ \langle Z^2\rangle = 1.3.$$

Fully catalyzed D-D cycle

In the event of the fully catalyzed cycle, the triple Lawson criterion fits similar concentrations of D and ^{3}He which could be expected in advance proceeding from the energy considerations:

$$n\tau T = 6.42\cdot 10^{16}\ \text{cm}^{-3}\cdot\text{s}\cdot\text{keV}\ (T = 54\ \text{keV},$$
$$n_D/n = n_{He}/n = 0.46,\ \delta = 10.87).$$

The concentration of plasma ions and its charge characteristics are as follows:

$$n_D = 0.46n,\ n_{He} = 0.46n,\ n_T = 0.0029n,$$
$$n_\alpha = 0.0386n,\ n_p = 0.0386n,$$
$$\langle Z\rangle = 1.5,\ \langle Z^2\rangle = 2.5.$$

The Lawson criterion that fits the above parameters n_D and δ has the form:

$$n\tau = 8.35\cdot 10^{14}\ \text{cm}^{-3}\cdot\text{s}\ (T = 123\ \text{keV}).$$

For comparison, we shall present the minimum value of parameter $n\tau T$ with $n_D/n = 0.5$:

$$n\tau T = 8.02\cdot 10^{16}\ \text{cm}^{-3}\cdot\text{s}\cdot\text{keV}\ (T = 56\ \text{keV},\ \delta = 7.3).$$

$$n_D = 0.5n,\ n_{He} = 0.313n,\ n_T = 0.0036n,$$
$$n_\alpha = 0.0845n,\ n_p = 0.0845n,$$

$$\langle Z\rangle = 1.4,\ \langle Z^2\rangle = 2.2.$$

The obtained results are close to those presented in Wurzel and Hsu 2022 both for the Lawson criterion

$$n\tau = 6.20\cdot 10^{14}\ \text{cm}^{-3}\cdot\text{s}\ (T = 106\ \text{keV}),$$

and for the triple criterion

$$n\tau T = 5.20\cdot 10^{16}\text{см}^{-3}\cdot\text{s}\cdot\text{keV}\ (T = 68\ \text{keV}).$$

However, the direct comparison is hard to make since no deuterium and ^{3}He concentrations are given in Wurzel and Hsu 2022.

Non-catalyzed D-D cycle for equimolar D-^{3}He fuel ($n_D = 0.5n$)

$n\tau = 6.01\cdot 10^{15}\text{cm}^{-3}\cdot\text{s}$ ($T = 103$ keV), $n\tau T = 4.89\cdot 10^{16}\text{cm}^{-3}\cdot\text{s}\cdot\text{keV}$ ($T = 67$ keV).

Conclusions

The key result from the above consideration is the determination (as part of a model problem) of the Lawson criterion and the triple Lawson criterion for D-^{3}He fueled thermonuclear devices with magnetic plasma confinement based on a refined temperature dependence of the D + ^{3}He → p + ^{4}He fusion reaction rate found using a new parametrization of the reaction cross-section and astrophysical factor in the effective radius approximation. The calculated reaction rate values are in a good agreement with the R-matrix theory results and the data contained in the NACRE II library, but, unlike these approaches, the effective radius approximation does not require an extensive computational power. It was found that in the event of a fully catalyzed cycle, the triple Lawson criterion fits equimolar D-^{3}He fuel, ($n_D/n = n_{He}/n = 0.46$), with the following parameters: $n\tau T = 6.42 \cdot 10^{16}$ cm^{-3}·s·keV ($T = 54$ keV), and is characterized by the smallest possible relative concentration of impurities. It is suggested that the developed approach to investigating the performance of particular thermonuclear systems with magnetic plasma confinement be used at the next stage.

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Appendix 1

Let us consider an exoergic reaction involving the formation of particles 1 and 2 with rest energies E01 and $E_{02}$and energy yield Q. We assume that the ingoing channel was characterized by a vanishingly small total momentum as is in the event of fusion reactions. Then the momentums of particles 1 and 2 have practically the same modules. We shall use a relativistic relation between the momentum and the energy, $p^2c^2 = E^2 - E_0^{\,2} = T^2 + 2TE_0$, $T = E–E_0$ is the kinetic energy of the particle. This leads to a system of two equations:

$$T_1^{\,2} + 2T_1E_{01} = T_2^{\,2} + 2T_2E_{02} \text{ and } T_1 + T_2 = Q. \quad \text{(A.1)}$$

The solution has the form:

$$T_1 = \frac{QE_{02}}{E_{01}+E_{02}} + \frac{0.5Q^2(E_{01}-E_{02})}{(E_{01}+E_{02}+Q)(E_{01}+E_{02})}. \quad \text{(A.2)}$$

The first term in (A.2) fits the nonrelativistic approximation:

$$T_1^{\,\text{nonrel}} = QE_{02}/(E_{01} + E_{02}) = Qm_2/(m_1 + m_2).$$

If particle 1 is a neutron and particle 2 is an alpha particle, the mass of which is assumed to be equal to 4.001506 a.m.u. as recommended by the CODATA system of physical constants (Tiesinga et al. 2021), then $T_1^{\,\text{nonrel}} = 14.048$ MeV.

The correction to the first term in (A.2) is negative and constitutes a fraction of it

$$\eta = \frac{0.5Q^2(E_{01} - E_{02})}{(E_{01}+E_{02} + Q)(E_{01}+E_{02})} \approx -1.4 \cdot 10^{-3}.$$

The neutron energy is therefore equal to $T_1 = 14.048(1 + \eta) = 14.028$ MeV.