





Research Article

Application of small perturbation theory for assessing variations of prompt neutron lifetime in a lead-cooled fast reactor^{*}

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Abstract

The paper considers the applicability of small perturbation theory to assessing the variations of the prompt neutron lifetime caused by variations in the isotope composition of a lead-cooled fast reactor. The generalized small perturbation theory formulas have been developed to calculate derivatives of the prompt neutron lifetime regarded as a bilinear neutron flux and neutron worth ratio. A numerical algorithm has been proposed for the step-by-step application of the small perturbation theory formulas to assess the prompt neutron lifetime variations caused by a major perturbation in the reactor isotope composition, e.g. by the complete change of the material used earlier as the neutron reflector. The advantage of the proposed approach has been shown which consists in that it is basically possible to determine the role of different neutron reactions, isotopes and energy groups in and their contributions to the total prompt neutron lifetime variation caused by major changes in the reactor isotope composition.

Keywords

small perturbation theory, bilinear ratios, prompt neutron lifetime, contributions of neutron reactions

Introduction

Neutronic and optimization calculations of nuclear reactors involve the need for comparing a variety of alternative designs in terms of multiple parameters that are important to ensuring efficient and safe operation of the reactor. These parameters include effective multiplication factor, breeding ratio, temperature and density reactivity coefficients, prompt neutron lifetime, effective fraction of delayed neutrons, etc. Some of these parameters can be presented as bilinear functionals of the neutron flux density of the following type:

$$J = \frac{\{a(r,E) \cdot \varphi(r,E)\}}{\{b(r,E) \cdot \varphi(r,E)\}}$$

where a(r, E) and b(r, E) are the kernels of the functionals; and $\varphi(r, E)$ is the energy spatial neutron flux

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distribution. Here and hereinafter, the curly brackets show the integration across the variables domain. Bilinear functionals include power peaking factors and breeding ratios.

Some of the other parameters may have the form of bilinear functionals of the neutron flux density and worth as shown below:

$$J = \frac{\{\varphi^+(r,E) \cdot a(r,E) \cdot \varphi(r,E)\}}{\{\varphi^+(r,E) \cdot b(r,E) \cdot \varphi(r,E)\}}$$

where $\phi^{\dagger}(r, E)$ is the energy spatial neutron worth distribution. Bilinear functionals include reactivity coefficients, prompt neutron lifetime, and effective fraction of delayed neutrons.

This paper deals with investigating the possibility for using small perturbation theory to estimate the prompt neutron lifetime variations as a bilinear functional of the neutron flux density and neutron worth in the event of major perturbations in the reactor isotope composition. An example was considered involving the complete change of the neutron reflector material in the course of which natural lead was changed for the material with a very small neutron absorption capacity. This is radiogenic lead with a high content of the ²⁰⁸Pb isotope.

Some publications (Apse et al. 2010; Kulikov et al. 2010; Shmelev et al. 2013; Kulikov et al. 2017) show that the use of radiogenic lead with a high content of lead-208 as coolant and the neutron reflector, instead of natural lead, leads to a substantial gain in the neutronic and thermophysical performance of a fast reactor. Besides, poor absorption of neutrons in physically thick radiogenic lead reflector is expected to lengthen greatly the prompt neutron lifetime and, therefore, increase, to a certain extent, the reactor resistance to fast power excursions.

An attempt was made in Apse et al.2012 to use generalized small perturbation theory to estimating the sensitivity of the prompt neutron lifetime to a minor change of the fast reactor isotope composition. It has been shown that the influence of a comparatively small (~ 10%) change in the concentrations of a fissionable isotope and lead coolant on the prompt neutron lifetime is estimated by generalized small perturbation theory with a sufficiently good accuracy (0.05–0.5%). This paper makes an attempt to use generalized small perturbation theory to estimate the influence of major reactor isotope composition perturbations (up to 100%) on the prompt neutron lifetime.

Formulas of small perturbation theory for bilinear functionals

The reactor flux density distribution equation can be written in the following operator form:

$$\widehat{L}(r,E) \cdot \varphi(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}(r,E) \cdot \varphi(r,E), \qquad (1)$$

where $\widehat{L}(r,E)$ is the operator that describes the neutron transport, absorption and moderation; $\widehat{Q}(r,E)$ is the operator that describes the fission neutron generation and K_{eff} is the effective multiplication factor.

For a unidimensional model of a nuclear reactor and multi-group diffusion approximation, equation (1) and its operators $\widehat{L}(r,E)$ and $\widehat{Q}(r,E)$ can be presented as follows:

$$\begin{aligned} -\frac{1}{r^{\alpha}} \cdot \frac{d}{dr} \left(r^{\alpha} \cdot D_{k}(r) \cdot \frac{d\varphi_{k}(r)}{dr} \right) + \left(\Sigma_{cfd,k}(r) + \omega_{z}^{2} \cdot D_{k}(r) \right) \cdot \varphi_{k}(r) = \\ &= \frac{\chi_{k}}{K_{cff}} \cdot \sum_{m} v \cdot \Sigma_{f,m}(r) \cdot \varphi_{m}(r) + \sum_{m=1}^{k-1} \Sigma^{m \to k}(r) \cdot \varphi_{m}(r) \end{aligned}$$

where α is the geometry indicator; k, m are the energy group numbers; D is the diffusion coefficient; $\varphi(r)$ is the neutron flux density; Σ_{cdf} is the neutron capture, fission and moderation cross-section; ω_z^2 is the geometrical parameter; χ is the fission neutron spectrum; $v \times \Sigma_f$ is the fission neutron generation macroscopic cross-section; and $\Sigma^{m \to k}$ is the neutron intergroup transition macroscopic cross-section.

According to small perturbation theory (Stumbur 1976), the sensitivity of the effective multiplication factor to the reactor isotope composition variations can be presented as the following bilinear functional (2):

$$\frac{\partial (1/K_{eff})}{\partial \rho_{l,i}} = \frac{\left\{\varphi^{+}(r,E) \cdot \frac{\partial \widehat{L}}{\partial \rho_{l,i}}\right\} - \frac{1}{K_{eff}} \cdot \left\{\varphi^{+}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{l,i}} \cdot \varphi(r,E)\right\}}{\{\varphi^{+}(r,E) \cdot \widehat{Q} \cdot \varphi(r,E)\}}$$

where ρ_{li} is the concentration of the *l* isotope in zone *i*.

The equation that describes the energy spatial distribution of the neutron worth is similar to equation (1) and differs from that in using Lagrange conjugate operators, i.e.

$$\widehat{L}^{+}(r,E) \cdot \varphi_{J}^{+}(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}^{+}(r,E) \cdot \varphi_{J}^{+}(r,E) - \frac{\partial J}{\partial \varphi} \qquad (3)$$

An important advantage of formula (2) is that it allows estimating the contribution of different processes (interzone and intergroup transitions of neutrons, and neutron absorption, moderation and multiplication), different isotopes and energy groups to the full value of the derivative (sensitivity) K_{eff} for the concentrations of isotopes. Unfortunately, small perturbation theory formulas make it possible to estimate variations K_{eff} with a good accuracy only with small perturbations in the reactor isotope composition.

There is a generalized small perturbation theory for linear and bilinear functionals that allows variations of functionals to be quantified with comparatively small changes of the isotope composition.

The key ideas of generalized small perturbation theory for bilinear functional (Khromov 1989, Kantorovich and Akilov 2004) are described in brief below.

General formalism suggests that an extended functional (Lagrangian operator) is built, in which equations describing the neutron flux density and value distributions are added to the sought-after functional through Lagrangian multipliers (-) [-, (-) (-) (-)

$$\begin{aligned} & \prod_{F} = \int \Phi \left\{ \phi_{J}(r,E) \cdot \left[\widehat{L} \cdot \varphi(r,E) - \frac{1}{K_{eff}} \cdot \widehat{Q} \cdot \varphi(r,E) \right] \right\} \\ & + \left\{ \phi_{J}(r,E) \cdot \left[\widehat{L}^{+} \cdot \varphi^{+}(r,E) - \frac{1}{K_{eff}} \cdot \widehat{Q}^{+} \cdot \varphi^{+}(r,E) \right] \right\} \end{aligned}$$

Equations can be obtained from the condition of the Lagrangian stationarity for Lagrange multipliers and the small perturbation theory formula. These equations have the following form:

$$\widehat{L}^{+}(r,E) \cdot \varphi_{J}^{+}(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}^{+}(r,E) \cdot \varphi_{J}^{+}(r,E) - \frac{\partial J}{\partial \varphi} \qquad (4)$$

$$\widehat{L}(r,E) \cdot \varphi_J(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}(r,E) \cdot \varphi_J(r,E) - \frac{\partial J}{\partial \varphi^+} \quad (5)$$

where the functional derivatives in the right-hand members can be calculated as follows:

$$\begin{aligned} \frac{\partial J}{\partial \varphi} &= J \cdot \left\{ \frac{a(r,E) \cdot \varphi^+(r,E)}{\{\varphi^+(r,E) \cdot a(r,E) \cdot \varphi(r,E)\}} - \frac{b(r,E) \cdot \varphi^+(r,E)}{\{\varphi^+(r,E) \cdot b(r,E) \cdot \varphi(r,E)\}} \right\} \\ \frac{\partial J}{\partial \varphi^+} &= J \cdot \left\{ \frac{a(r,E) \cdot \varphi^+(r,E)}{\{\varphi^+(r,E) \cdot a(r,E) \cdot \varphi(r,E)\}} - \frac{b(r,E) \cdot \varphi^+(r,E)}{\{\varphi^+(r,E) \cdot b(r,E) \cdot \varphi(r,E)\}} \right\} \end{aligned}$$

It can be easily shown that the functional derivatives are orthogonal with respect to the solutions of respective homogeneous equations, that is

$$\left\{ \frac{\partial J}{\partial \varphi(r,E)} \cdot \varphi(r,E) \right\} = 0$$
$$\left\{ \frac{\partial J}{\partial \varphi^{+}(r,E)} \cdot \varphi^{+}(r,E) \right\} = 0$$

According to the Fredholm alternative, it follows from this that solutions exist for heterogeneous equations (4) and (5). This results in the following two systems of homogeneous and heterogeneous equations.

Homogeneous equations:

$$\widehat{L}(r,E) \cdot \varphi(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}(r,E) \cdot \varphi(r,E),$$

$$\widehat{L}^{+}(r,E) \cdot \varphi^{+}(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}^{+}(r,E) \cdot \varphi^{+}(r,E)$$
(6)

Heterogeneous equations:

$$\widehat{L}(r,E) \cdot \varphi_J(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}(r,E) \cdot \varphi_J(r,E) - \frac{\partial J}{\partial \varphi^+},$$
(7)
$$\widehat{L}^+(r,E) \cdot \varphi_J(r,E) = \frac{1}{K_{eff}} \cdot \widehat{Q}^+(r,E) \cdot \varphi_J^+(r,E) - \frac{\partial J}{\partial \varphi}$$

These systems need to be solved successively since the functional derivatives in the right-hand members of heterogeneous equations (6) can be determined only by finding the solution to homogeneous equations (7).

If the systems of homogeneous and heterogeneous equations (6) and (7) are solved, the Lagrangian variance caused, i.e. by the variance of the l isotope concentration in zone i, can be calculated as follows:

$$\delta F = J \cdot \left[\frac{\left\{ \varphi^{+}(r,E) \cdot \frac{\partial a(r,E)}{\partial \rho_{l,i}} \cdot \varphi(r,E) \right\}}{\left\{ \varphi^{+}(r,E) \right\} \cdot a(r,E) \cdot \varphi(r,E)} - \left\{ \frac{\left\{ \varphi^{+}(r,E) \cdot \frac{\partial b(r,E)}{\partial \rho_{l,i}} \cdot \varphi(r,E) \right\}}{\left\{ \varphi^{+}(r,E) \right\} \cdot b(r,E) \cdot \varphi(r,E)} \right]} \right] \cdot \delta \rho_{l,i} + \left[\left\{ \varphi^{+}_{J}(r,E) \cdot \frac{\partial \widehat{L}}{\partial \rho_{l,i}} \cdot \varphi(r,E) \right\} + \left\{ \varphi^{+}_{J}(r,E) \cdot \frac{\partial \widehat{L}}{\partial \rho_{l,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \delta \rho_{l,i} - \left\{ \varphi^{+}_{J}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{l,i}} \cdot \varphi(r,E) \right\} + \left[\cdot \frac{\delta \rho_{l,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{l,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{l,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{l,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{l,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{l,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{J}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{I}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{I}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} \cdot \varphi_{I}(r,E) \right\} + \left[\cdot \frac{\delta \rho_{I,i}}{K_{eff}} - \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}} + \left\{ \varphi^{+}_{I}(r,E) \cdot \frac{\partial \widehat{Q}}{\partial \rho_{I,i}}$$

It was proved in the functional analysis theory (Khromov 1989; Kantorovich and Akilov 2004) that the Lagrangian operator F at the stationary point is equal to the sought-after functional J, the first-order variance of the Lagrangian operator F is equal to the variance of the first-order functional J, and the uncertainty of the functional J has the second order of smallness with respect to the uncertainties in defining the functions $\varphi(r, E)$, $\varphi^+(r, E)$.

Therefore, it is expression (8) that is the small perturbation theory formula for the sought-after bilinear functional.

Use of small perturbation theory for estimating prompt neutron lifetime variations

The described procedure for calculating the sensitivity of bilinear functionals was put in the TIME26 code (Apse and Shmelev 2008). Test calculations were undertaken for a unidimensional spherical model of a lead-cooled fast reactor (Orlov et al. 2000) with lead reflector. The geometrical model included three reactor cores with the same content of the plutonium fraction in mixed uranium-plutonium nitride fuel but with different fuel rod diameters, and a lead side shield. The prompt neutron lifetime was considered as the bilinear functional:

$$l_{p} = \frac{\left\{\varphi^{+}(r,E) \cdot diag(\frac{1}{V}) \cdot \varphi(r,E)\right\}}{\{\varphi^{+}(r,E) \cdot \widehat{Q}(r,E) \cdot \varphi(r,E)\}}$$

The key objective of the calculations was to estimate the variation of this functional with the complete change of the side shield material (natural lead) for lead-208 using small perturbation theory formula (8). The prompt neutron lifetime is specific as a bilinear functional in that the whole of the contribution to its variation is provided only by the terms in expression (8) that describe the neutron leakage, absorption and moderation processes. The small perturbation theory formula (8) can be used to produce the prompt neutron lifetime derivative l_p from ρ_{l_i} , that is, from the concentration of the *l* isotope in zone *i*:

$$\frac{\partial I_p}{\partial \rho_{I,i}} = \left[\left\{ \varphi_J^+(r,E) \cdot \frac{\partial \widehat{L}}{\partial \rho_{I,i}} \cdot \varphi(r,E) \right\} + \left\{ \varphi^+(r,E) \cdot \frac{\partial \widehat{L}}{\partial \rho_{I,i}} \cdot \varphi(r,E) \right\} \right] (9)$$

In more details, the components in expression (9) can be written as:

$$\begin{split} &\left\{\varphi_{J}^{+}(r,E)\cdot\frac{\partial\widehat{L}}{\partial\rho_{l,i}}\cdot\varphi(r,E)\right\} = \left\{\nabla\varphi_{J}^{+}(r,E)\cdot\frac{\partial\widehat{D}}{\partial\rho_{l,i}}\cdot\nabla\varphi(r,E)\right\} + \\ & \omega_{z}^{2}\cdot\left\{\varphi_{J}^{+}(r,E)\cdot\frac{\partial\widehat{D}}{\partial\rho_{l,i}}\cdot\varphi(r,E)\right\} + \left\{\varphi_{J}^{+}(r,E)\cdot\frac{\partial\widehat{\Sigma}}{\partial\rho_{l,i}}\cdot\varphi(r,E)\right\} \\ &\left\{\varphi^{+}(r,E)\cdot\frac{\partial\widehat{L}}{\partial\rho_{l,i}}\cdot\varphi_{J}(r,E)\right\} = \left\{\nabla\varphi^{+}(r,E)\cdot\frac{\partial\widehat{D}}{\partial\rho_{l,i}}\cdot\nabla\varphi_{J}(r,E)\right\} + \\ & \omega_{z}^{2}\cdot\left\{\varphi^{+}(r,E)\cdot\frac{\partial\widehat{D}}{\partial\rho_{l,i}}\cdot\varphi_{J}(r,E)\right\} + \left\{\varphi^{+}(r,E)\cdot\frac{\partial\widehat{\Sigma}}{\partial\rho_{l,i}}\cdot\varphi_{J}(r,E)\right\} \end{split}$$

By summing up the components in these expressions, one can identify the contributions of individual processes to the prompt neutron lifetime derivative:

 contribution of the neutron radial interzone transitions and leakage:

$$\frac{\partial I_p}{\partial \rho_{l,i}}(j_r) = \left\{ \nabla \varphi_J^+(r,E) \cdot \frac{\partial \widehat{D}}{\partial \rho_{l,i}} \cdot \nabla \varphi(r,E) \right\} + \left\{ \nabla \varphi^+(r,E) \cdot \frac{\partial \widehat{D}}{\partial \rho_{l,i}} \cdot \nabla \varphi_J(r,E) \right\}$$

- contribution of the axial neutron leakage:

$$\frac{\partial l_z}{\partial \rho_{l,i}}(j_r) = \omega_z^2 \cdot \left\{ \varphi_J^+(r,E) \cdot \frac{\partial \widehat{D}}{\partial \rho_{l,i}} \cdot \varphi(r,E) \right\} + \omega_z^2 \cdot \left\{ \varphi^+(r,E) \cdot \frac{\partial \widehat{D}}{\partial \rho_{L,i}} \cdot \varphi_J(r,E) \right\}$$

- contribution of the neutron absorption:

$$\frac{\partial I_z}{\partial \rho_{I,i}} (\Sigma_c) = \left\{ \varphi_J^+(r,E) \cdot \sigma_{c,I}(E) \cdot \varphi(r,E) \right\} + \left\{ \varphi^+(r,E) \cdot \sigma_{c,I}(E) \cdot \varphi_J(r,E) \right\}$$

contribution of the neutron moderation (spectral effect):

$$\frac{\partial I_z}{\partial \rho_{l,i}} (\Sigma_d) = \sum_m \sigma_{d,k,l} \cdot \int_{\Delta R_i}^{\omega^{\alpha}} r^{\alpha} \cdot dr \cdot \begin{bmatrix} \varphi_{J,k}^+(r) \cdot \varphi_k(r) + \\ \varphi_k^+(r) \cdot \varphi_{J,k}(r) \end{bmatrix}$$
$$- \sum_{m=1}^{k-1} \sigma_l^{m \to k} \cdot \int_{\Delta R_i}^{\omega^{\alpha}} r^{\alpha} \cdot dr \cdot \begin{bmatrix} \varphi_{J,k}^+(r) \cdot \varphi_m(r) + \\ \varphi_k^+(r) \cdot \varphi_{J,m}(r) \end{bmatrix}$$

If desired, expression (9) can be detailed further and the contributions of individual geometrical zones and energy groups to the prompt neutron lifetime derivative can be determined. Fig. 1 presents the curves that characterize the prompt neutron lifetime prolongation as the result of the gradual natural lead change for lead-208 in the neutron reflector with a variable thickness.



Figure 1. Prompt neutron lifetime as a function of the ²⁰⁸Pb fraction in the Pb reflector.

It can be seen from the above curves that the most intensive growth in the prompt neutron lifetime l_p takes place at the closing change stage when the final percentage of natural lead is displaced by lead-208. This is expressed especially graphically in the thick (3 m) reflector design. If the fraction of lead-208 is increased from zero to 20% at the initial stage, then l_p increases from 3 µs to only 4 µs. And if the fraction of lead-208 grows from 80 to 100% at the closing stage, than l_p increases from 33 µs to 705 µs. With the five remaining natural lead percent, the lifetime grows nearly fourfold, from 180 µs with 95% of lead-208 to 705 µs with 100% of lead-208.

The explanation is that natural lead as such contains much lead-208 (52.4%). Therefore, the full content of lead-208 in the reflector increases insignificantly at the initial change stage. It is only at the closing stage, when the fraction of lead-208 tends to 100%, that the l_p prolongation is the greatest thanks to its unique neutronic properties (very small absorption and moderation of neutrons).

The use of small perturbation theory (9) makes it possible to assess the role played by the neutron interzone transitions, absorption and moderation in the growth of l_p . Unfortunately, small perturbation theory is called so exactly because it allows predicting variations in the sought-after functionals and estimating the contributions of different processes to these variations with rather small variances of controlling parameters. The complete change of the reflector material cannot be called a small perturbation. Actually, as can be seen in Fig. 1, the complete change of natural lead for lead-208 in the thick (3 m) reflector prolongs the prompt neutron lifetime from 3 μ s to 705 μ s, that is, by a factor of more than 200. A possibility however exists for overcoming this difficulty. The process of substituting natural lead for lead-208 can be broken down into such number of stages within each of which the small perturbation theory formulas predict the variation of l_p and estimate the contributions of different processes to this variation with an acceptable accuracy. This opens up the possibility for tracing, in a step-by-step manner, the evolution of variations in the contributions of different processes to the growth of l_p with the gradual substitution of natural lead for lead-208. Further, by summing up these stepwise contributions, one can estimate the total contribution of different processes to the overall prolongation of l_p even with major isotope composition perturbations.

Figs 2, 3 show the evolution of the contributions from interzone transitions, absorption and moderation to the prompt neutron lifetime prolongation with the gradual substitution of natural lead for lead-208 in the thin (0.5 m) and thick (1 m) reflector. It can be seen that the contribution of smaller neutron absorption in a comparatively thin reflector plays a key rather than a dominant role. The contribution of smaller absorption becomes rapidly the



Figure 2. Contributions of neutron leakage, absorption and moderation to lifetime prolongation (reflector thickness 0.5 m).



Figure 3. Contributions of neutron leakage, absorption and moderation to lifetime prolongation (reflector thickness 1 m).

dominant one when the reflector thickness is increased from 0.5 m to 1 m. It exceeds 100% at the closing stage since the changes in the interzone transitions and moderation (the spectral effect) are minor but shorten the prompt neutron lifetime.

By summing up the neutron lifetime prolongations for stages of the natural lead gradual substitution for lead-208, one can estimate the contributions of the neutron radial interzone transition, absorption and moderation processes to the complete neutron lifetime prolongation. These data are presented in Table 1.

Table 1. Contributions of the neutron interzone transition, absorption and moderation to the lifetime prolongation.

Reflector thickness, cm	$\Delta l_{\rm p}({\rm j_r}),\%$	$\Delta l_{\rm p}(\Sigma_{\rm c}), \%$	$\Delta l_{\rm p}(\Sigma_{\rm d}), \%$
50	17.9	51.6	30.5
100	1.6	97.4	1.0
150	0.09	101.23	-1.32
200	-0.003	100.607	-0.604

It can be seen from these results that, as the reflector thickness increases, the role of very small neutron absorption by lead-208 increases and becomes rapidly dominant. Interestingly, the cumulative action of the interzone transitions and the spectral effect is insignificant but shortens the prompt neutron lifetime as the reflector thickness is increased.

Conclusions

A stepwise algorithm has been proposed for using small perturbation theory formulas to estimate the sensitivity of the prompt neutron lifetime to major changes in the nuclear reactor isotope composition. The applicability of such approach has been considered for estimating the change in the prompt neutron lifetime caused by the complete substitution of the reflector material in a lead-cooled fast reactor (natural lead was substituted by radiogenic lead with the dominant fraction of the lead-208 isotope). The gradual stepwise substitution of the reflector material has shown that it is possible to use the generalized small perturbation theory formulas at each step and obtain the final result with a good accuracy (with complete substitution of the reflector). The key advantage of the proposed approach is its capability to identify the role and estimate the contribution of all neutron processes, isotopes and energy groups to the complete change of the prompt neutron lifetime.

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